Modeling & Problem Solving

IE 101
Fall 2008
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Department of Industrial Engineering

Industrial Engineering (IE)

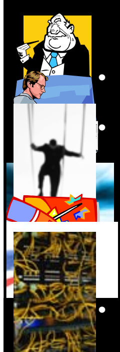
The Industrial Revolution of the 19th century probably did more to shape life in the modern industrialized world than any event in history. Large factories with mass production created a need for managing them effectively and efficiently.

The field of Decision Science (DS) also known as Management Science (MS), Systems Science (SS), Operations Research (OR) in a more general sense, started with the publication of The Principles of Scientific Management in 1911 by Frederick W. Taylor, the father of IE.

His approach relied on the measurement of industrial productivity and on time /movement studies in the factories. The goal of his scientific management was to determine the best method for performing tasks in the least amount of time.

Nowadays, the IE=OR/MS/DS/SS approach has been providing assistance to managers in developing the expertise and tools necessary to understand the decision problems, put them in analytical terms and then solve them.

Decision Environment



- The decision-maker refers to an individual, not a group.
- The IE analyst who models the problem in order to help the decision maker,
  Controllable factors (including your personal abilities and physical resources),
  Uncontrollable factors,
  The possible outcomes of the decision,
  The environment/structural constraints
- Dynamic interactions among these components.

STEPS: Effective Decision Making

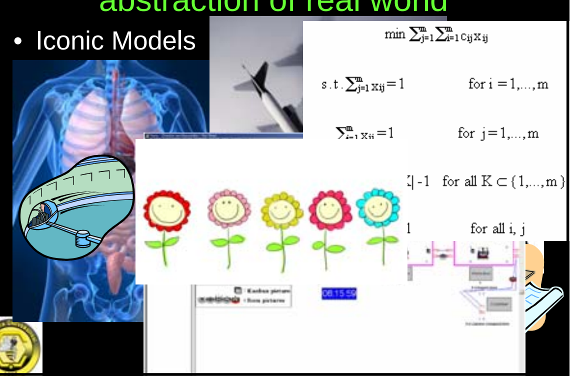
1. Understanding the Problem: It is critical for a good decision maker to clearly understand the probl m, the objective, and the constraints involved.
2. Constructing a Mod volves the "translation" of the problem into precise athem cal language in order to make calculations and comparison of the outcomes under different possible scenarios.
3. Finding a Good Solution: It is important here to choose the proper solving technique, depending on the specific characteristics of the model. After the model is solved, validation of the obtained results must be do in order to avoid an unrea solution.
4. Communic e Results with the De n-Maker: The results obtained OR/MS/DS/SS anal e to be properly communicate the decision-make is the "sale" part. If the decision-make not buy the IE st recommendations, he/she will not im ment any of the .
5. Implementation: Your decision means nothing unless you put it into action. A decision without a plan of action is a daydream.

S1:Understand the problem

- Problem understanding encompasses a problem structure, and a diagnostic process to assist us in problem formulation (i.e., giving a Form to a complex situation) and representation. This stage is the most important aspect of the decision-making process. Problem understanding is an interactive process between the decision maker and the IE. The decision maker may be unfamiliar with the analytic details of the problem formulation such as what elements to include in the model, and how to include them as variables, constraints, etc.
- Since the strategic solution to any problem involves making certain assumptions, it is necessary to determine the extent to which the strategic solution changes when the assumptions change. You will learn this by performing the "what-if" scenarios and the necessary sensitivity analysis.
- Gathering reliable information at the right time is a component of good decisions. It is helpful to understand the nature of the problem by asking "who?", "what?", "why?", "when", "where" and "how?". Finally, break them into three input groups, namely: Parameters, Controllable, and Uncontrollable inputs. Uncontrollable factors are the main components of decision-making which must be dealt with, by, e.g., forecasting.

S2: Model abstraction of real world

- Iconic Models



S2: Mathematical Models

- Most IE studies involve the construction of a mathematical model. The model is a collection of logical and mathematical relationships that represents aspects of the situation under study. Models describe important relationships between variables, include an objective function with which alternative solutions are evaluated, and constraints that restrict solutions to feasible values.
- Although the IE would hope to study the broad implications of the problem using a systems approach, a model cannot include every aspect of a situation. A model is always an abstraction that is of necessity simpler than the real situation. Elements that are irrelevant or unimportant to the problem are to be ignored, hopefully leaving sufficient detail so that the solution obtained with the model has value with regard to the original problem.
- Models must be both tractable, capable of being solved, and valid, representative of the original situation. These dual goals are often contradictory and are not always attainable. It is generally true that the most powerful solution methods can be applied to the simplest, or most abstract, model.

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S3: Solve → Optimize

- The IE has a wide variety of methods available for problem solving. For mathematical programming models there are optimization techniques appropriate for almost every type of problem, although some problems may be difficult to solve.
- For models that incorporate statistical variability there are methods such as probability analysis and simulation that estimate statistics for output parameters.
- In most cases the methods are implemented in computer programs. It is important that at least some member of an IE study team be aware of the tools available and be knowledgeable concerning their capabilities and limitations.

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S4: COMMUNICATE

- All IE concepts focus on communication of the results and recommended courses of actions (strategies). This helps all involved to build a consensus concerning the possible outcomes and recommended course of action. The decision-maker might incorporate some other perspectives of the problem, such as cultural, political, psychological, etc., into the management scientist's recommendations.
- At the "what-if" analysis stage of modeling, the modeler and the owner of the problem must concentrate on what can happen. The "what-if" analysis provides "look ahead" management. The management can use a dynamic model to experiment with future consequences of new policies. It provides information on what is likely to happen, not what necessarily will happen.
- The IE's main interest should be in providing assistance in decision-making and finding methods of solution that are more elegant or marginally faster than existing methods.
- Interaction between the decision-maker and IE must be open, interactive, and focused on the ultimate goal of the effort.

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S5: Implementation

- Verification is the process of comparing the computer code with the model to ensure that the code is a correct implementation of the model. During verification, one checks the computer implementation of the model.
- Validation is the process of comparing the model's output with the behavior of the phenomenon. This is to say that confirmation of the model's behavior is essential. How else can one determine if the proper model has been built. Then there is always the question of cost. Modeling can be very expensive. The more complicated the model, the greater the cost. Inputs and constraints added to existing problems create extra costs. Plus, there is the matter of timely decisions.
- Success is the ability to put into implementation phase what is in your decision model. In recent years, there has been increasing concern over the relevance of many aspects of the IE modeling process.

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AN EXAMPLE

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THE Traveling Salesman Problem

- The traveling salesman problem (TSP) asks for the shortest route to visit a collection of cities and return to the starting point.
- TSP is one which has commanded much attention of mathematicians and computer scientists specifically because it is so easy to describe and so difficult to solve.

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Traveling Salesman Problem

The problem can simply be stated as:

If a traveling salesman wishes to visit exactly once each of a list of *m* cities (where the cost of traveling from city *i* to city j is c_{ij}) and then return to the home city, what is the least costly route the traveling salesman can take?

Traveling Salesman Problem

Formulation:

To formulate the asymmetric TSP on m cities, one introduces zero-one variables:

$$x_{ij} = \begin{cases} 1 \text{ if the edge } i \rightarrow j \text{ is in the tour} \\ 0 \text{ otherwise} \end{cases}$$

and given the fact that every node of the graph must have exactly one edge pointing towards it and one pointing away from it, one obtains the classic assignment problem.

Traveling Salesman Problem

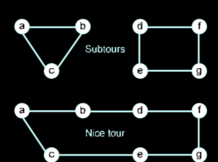
$$\min \sum_{j=1}^{m} \sum_{i=1}^{m} c_{ij} x_{ij}$$

$$\text{s.t.} \sum_{j=1}^{m} x_{ij} = 1 \qquad \text{for } i = 1,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad \text{for } j = 1,\ldots,m$$

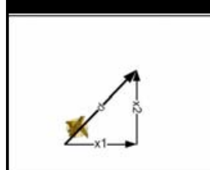
$$x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i, j$$

These constraints alone are not enough since this formulation would allow "subtours", that is, it would allow disjoint loops to occur.



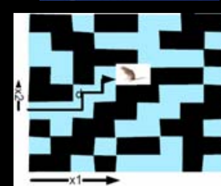
Cost coefficients=f(DISTANCEs)

Euclidean



$\|x\|_2 = (|x_1|^2 + \cdots + |x_n|^2)^{\frac{1}{2}}$

$\|x\|_1 = |x_1| + \cdots + |x_n|$

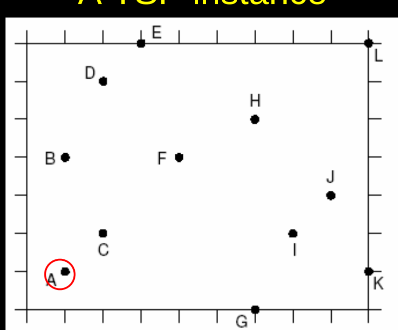


Manhattan

Tchbychev's

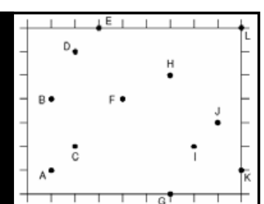
$\|x\|_\infty = \max\{|x_1|, \ldots, |x_n|\}$

A TSP instance



A TSP instance

$$A = \begin{bmatrix}1\\1\end{bmatrix}, B = \begin{bmatrix}1\\4\end{bmatrix}, C = \begin{bmatrix}2\\2\end{bmatrix},$$
$$D = \begin{bmatrix}2\\6\end{bmatrix}, E = \begin{bmatrix}3\\7\end{bmatrix}, F = \begin{bmatrix}4\\4\end{bmatrix},$$
$$G = \begin{bmatrix}6\\0\end{bmatrix}, H = \begin{bmatrix}6\\5\end{bmatrix}, I = \begin{bmatrix}7\\2\end{bmatrix},$$
$$J = \begin{bmatrix}8\\3\end{bmatrix}, K = \begin{bmatrix}9\\1\end{bmatrix}, L = \begin{bmatrix}9\\7\end{bmatrix}.$$



A TSP instance

$$A=\begin{bmatrix}1\\1\end{bmatrix}, B=\begin{bmatrix}1\\4\end{bmatrix}, C=\begin{bmatrix}2\\2\end{bmatrix}, D=\begin{bmatrix}2\\6\end{bmatrix}, E=\begin{bmatrix}3\\7\end{bmatrix}, F=\begin{bmatrix}4\\4\end{bmatrix}, G=\begin{bmatrix}6\\0\end{bmatrix}, H=\begin{bmatrix}6\\5\end{bmatrix}, I=\begin{bmatrix}7\\2\end{bmatrix}, J=\begin{bmatrix}8\\3\end{bmatrix}, K=\begin{bmatrix}9\\1\end{bmatrix}, L=\begin{bmatrix}9\\7\end{bmatrix}.$$

l_1	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	2	6	8	6	6	9	7	9	8	14
B	3	0	3	3	5	3	9	6	8	8	11	11
C	2	3	0	4	6	4	6	7	5	7	8	12
D	6	3	4	0	2	4	10	5	9	9	12	8
E	8	5	6	2	0	4	10	5	9	9	12	6
F	6	3	4	4	4	0	6	3	5	5	8	8
G	6	9	6	10	10	6	0	5	3	5	4	10
H	9	6	7	5	5	3	5	0	4	4	7	5
I	7	8	5	9	9	5	3	4	0	2	3	7
J	9	8	7	9	9	5	5	4	2	0	3	5
K	8	11	8	12	12	8	4	7	3	3	0	6
L	14	11	12	8	6	8	10	5	7	5	6	0

$\|x\|_1 = |x_1| + \cdots + |x_n|$

Manhattan

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A TSP instance

$$A=\begin{bmatrix}1\\1\end{bmatrix}, B=\begin{bmatrix}1\\4\end{bmatrix}, C=\begin{bmatrix}2\\2\end{bmatrix}, D=\begin{bmatrix}2\\6\end{bmatrix}, E=\begin{bmatrix}3\\7\end{bmatrix}, F=\begin{bmatrix}4\\4\end{bmatrix}, G=\begin{bmatrix}6\\0\end{bmatrix}, H=\begin{bmatrix}6\\5\end{bmatrix}, I=\begin{bmatrix}7\\2\end{bmatrix}, J=\begin{bmatrix}8\\3\end{bmatrix}, K=\begin{bmatrix}9\\1\end{bmatrix}, L=\begin{bmatrix}9\\7\end{bmatrix}.$$

l_2	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	$\sqrt{2}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{18}$	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{37}$	$\sqrt{53}$	8	10
B	3	0	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$	3	$\sqrt{41}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{50}$	$\sqrt{73}$	$\sqrt{73}$
C	$\sqrt{2}$	$\sqrt{5}$	0	4	$\sqrt{26}$	$\sqrt{8}$	$\sqrt{20}$	5	5	$\sqrt{37}$	$\sqrt{50}$	$\sqrt{74}$
D	$\sqrt{26}$	$\sqrt{5}$	4	0	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{40}$	$\sqrt{17}$	$\sqrt{41}$	$\sqrt{45}$	$\sqrt{74}$	$\sqrt{50}$
E	$\sqrt{40}$	$\sqrt{13}$	$\sqrt{26}$	$\sqrt{2}$	0	$\sqrt{10}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{72}$	6
F	$\sqrt{18}$	3	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{10}$	0	$\sqrt{20}$	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{34}$	$\sqrt{34}$
G	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{20}$	$\sqrt{40}$	$\sqrt{58}$	$\sqrt{20}$	0	5	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{10}$	$\sqrt{58}$
H	$\sqrt{41}$	$\sqrt{26}$	5	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{5}$	5	0	$\sqrt{10}$	$\sqrt{8}$	5	$\sqrt{13}$
I	$\sqrt{37}$	$\sqrt{40}$	5	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{13}$	$\sqrt{5}$	$\sqrt{10}$	0	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{29}$
J	$\sqrt{53}$	$\sqrt{50}$	$\sqrt{37}$	$\sqrt{45}$	$\sqrt{41}$	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{8}$	$\sqrt{2}$	0	$\sqrt{5}$	$\sqrt{17}$
K	8	$\sqrt{73}$	$\sqrt{50}$	$\sqrt{74}$	$\sqrt{72}$	$\sqrt{34}$	$\sqrt{10}$	5	$\sqrt{5}$	$\sqrt{5}$	0	6
L	10	$\sqrt{73}$	$\sqrt{74}$	$\sqrt{50}$	6	$\sqrt{34}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{29}$	$\sqrt{17}$	6	$\sqrt{0}$

Euclidean

$\|x\|_2 = (|x_1|^2 + \cdots + |x_n|^2)$

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A TSP instance

$$A=\begin{bmatrix}1\\1\end{bmatrix}, B=\begin{bmatrix}1\\4\end{bmatrix}, C=\begin{bmatrix}2\\2\end{bmatrix}, D=\begin{bmatrix}2\\6\end{bmatrix}, E=\begin{bmatrix}3\\7\end{bmatrix}, F=\begin{bmatrix}4\\4\end{bmatrix}, G=\begin{bmatrix}6\\0\end{bmatrix}, H=\begin{bmatrix}6\\5\end{bmatrix}, I=\begin{bmatrix}7\\2\end{bmatrix}, J=\begin{bmatrix}8\\3\end{bmatrix}, K=\begin{bmatrix}9\\1\end{bmatrix}, L=\begin{bmatrix}9\\7\end{bmatrix}.$$

l_∞	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	1	5	6	3	5	5	6	7	8	8
B	3	0	2	2	3	3	5	5	6	7	8	8
C	1	2	0	4	5	2	4	4	5	6	7	7
D	5	2	4	0	1	2	6	4	5	6	7	7
E	6	3	5	1	0	3	7	3	5	5	6	6
F	3	3	2	2	3	0	4	2	3	4	5	5
G	5	5	4	6	7	4	0	5	2	3	3	7
H	5	5	4	4	3	2	5	0	3	2	4	3
I	6	6	5	5	5	3	2	3	0	1	2	5
J	7	7	6	6	5	4	3	2	1	0	2	4
K	8	8	7	7	6	5	3	4	2	2	0	6
L	8	8	7	7	6	5	7	3	5	4	6	0

Tchbychev's

$\|x\|_\infty = \max\{|x_1|, \ldots, |x_n|\}$

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TSP is hard to solve: HEURISTIC

- CONSTRUCTION Heuristic
  - To obtain an initial solution
  - not as good as optimal
- IMPROVEMENT Heuristic
  - To improve the previous solution
  - near optimal

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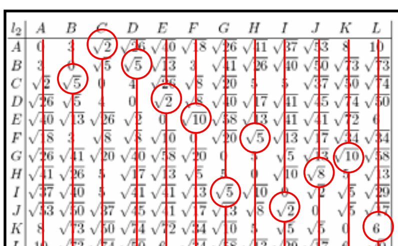
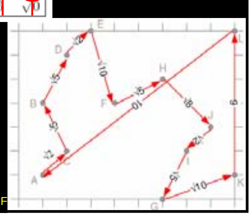


NEAREST NEIGHBOR
Next,
Go to the nearest unvisited city !

A → C → B → D → E → F → H → J → I → G(or K) → K → L → A

Initial tour length is 38.3399.



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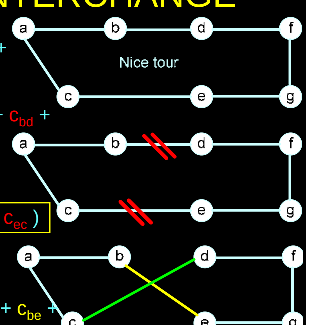
GAIN of an INTERCHANGE

Tour Length = $c_{ab} + c_{bd} + c_{df} + c_{fg} + c_{ge} + c_{ec} + c_{ca}$

OLD Tour Length = $c_{ab} + c_{bd} + c_{df} + c_{fg} + c_{ge} + c_{ec} + c_{ca}$

NEW = OLD + $(c_{be} + c_{dc}) - (c_{bd} + c_{ec})$

NEW Tour Length = $c_{ab} + c_{be} + c_{fd} + c_{gf} + c_{eg} + c_{dc} + c_{ca}$



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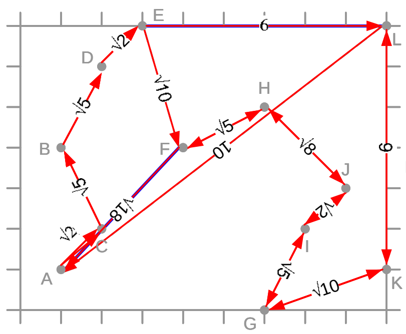
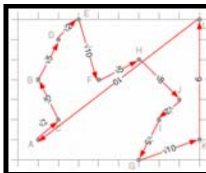


Tour(NN)=38.3399

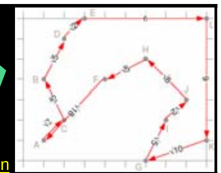
Delete (E, F) & (L,A)

Gain=+ 2.9196

Impr.Tour= 35.42026



Continue in this fashion



$GAIN$	(B,D)	(D,E)	(E,F)	(F,H)	(H,I)	(I,J)	(J,K)	(K,G)	(G,L)	(L,A)
(A,C)	−3.35	−7.37	−4.58	−5.59	−8.24	−9.45	−6.90	−7.59	−9.19	
(C,B)		−3.96	−2.70	−3.46	−7.01	−8.76	−6.93	−7.62	−7.38	0.63
(B,D)			−1.04	−2.65	−6.74	−9.82	−9.06	−9.61	−7.38	−1.41
(D,E)				−2.78	−6.28	−10.28	−10.37	−11.12	−7.19	−1.98
(E,F)					−1.74	−5.43	−5.48	−7.12	−5.15	2.92
(F,H)						−3.64	−4.13	−4.07	−1.20	0.00
(H,I)							−1.70	−1.25	−0.29	1.94
(I,J)								−1.27	−0.21	1.21
(J,K)									−1.62	1.75
(K,G)										−2.45

Exercise

Do the same for

- Manhattan Distance
- Tchbychev's Distance