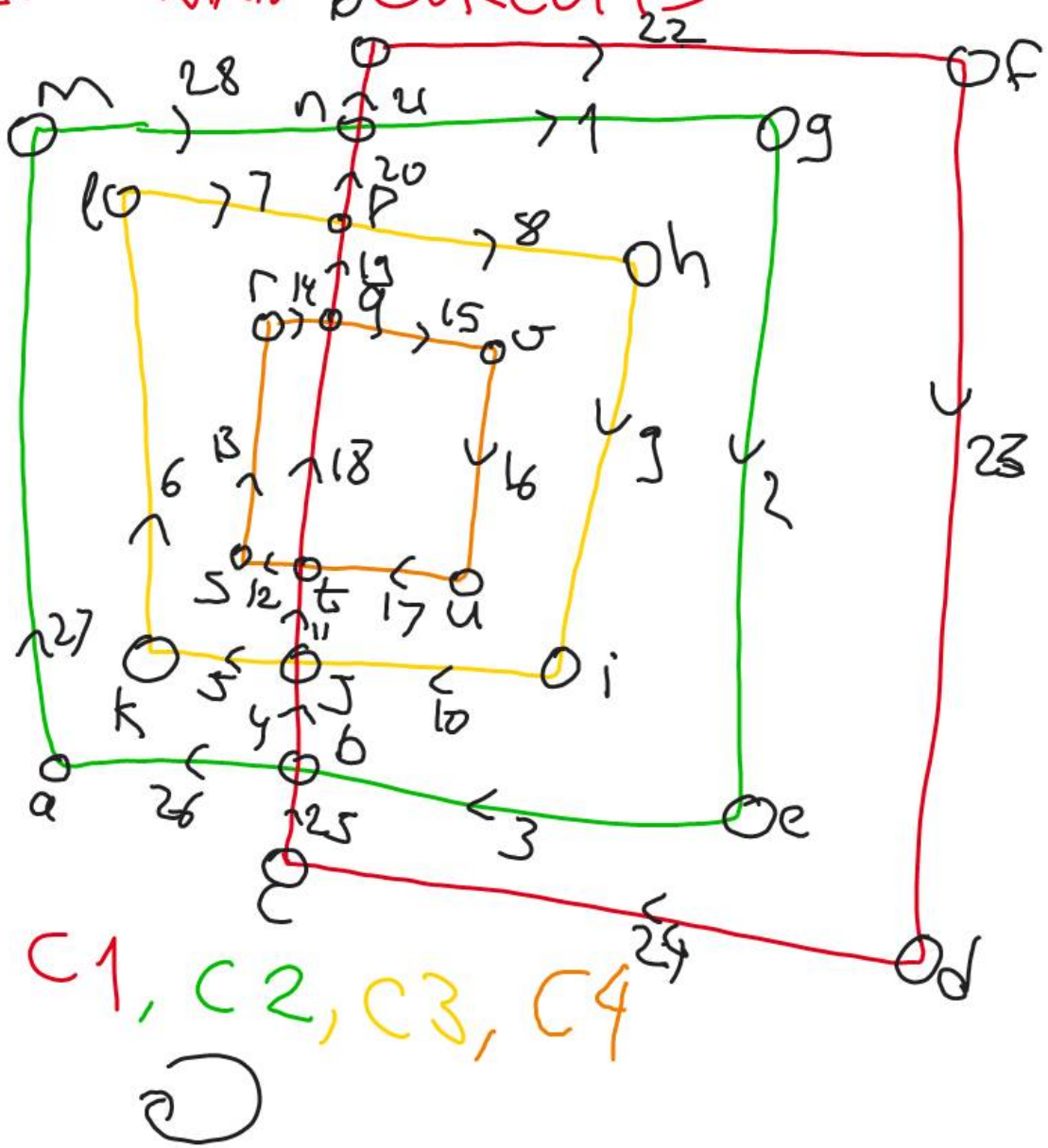
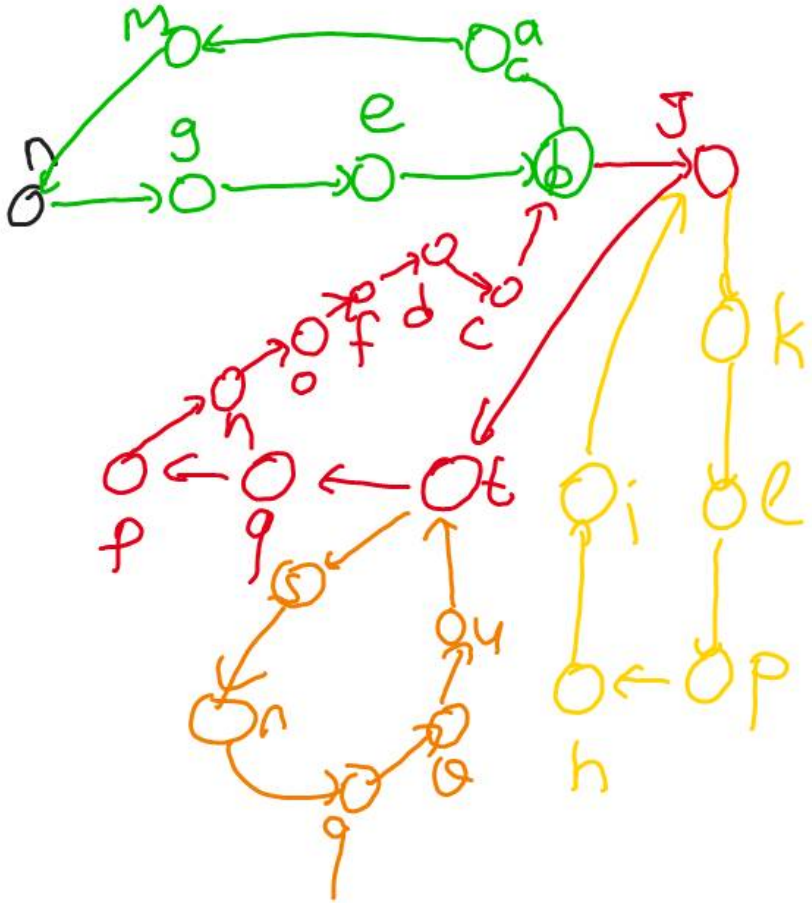
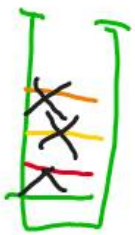
EULERIAN CIRCUITS







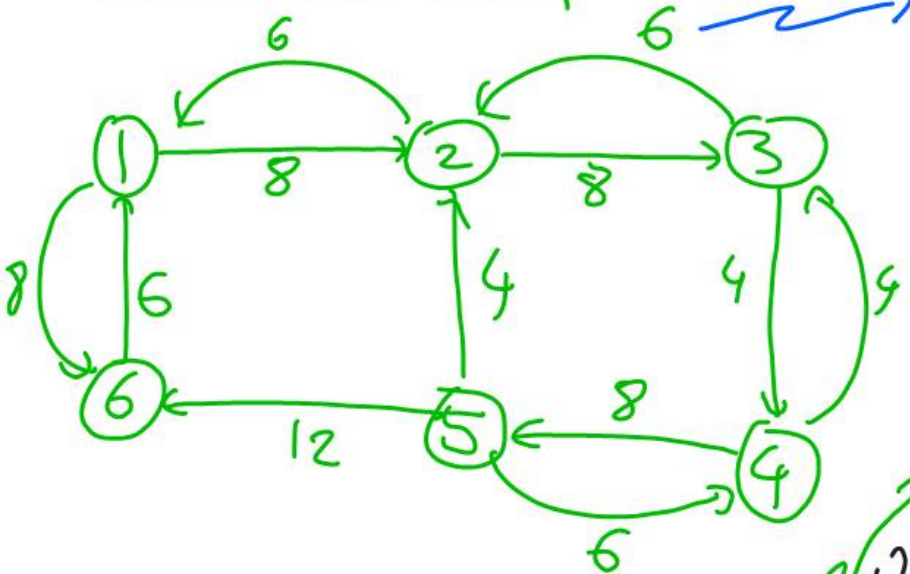
C1, C2, C3, C4

CHINESE POSTMAN PROBLEM

To make a tour visiting all the edges at least once

2→ Min value of edges visited more than once

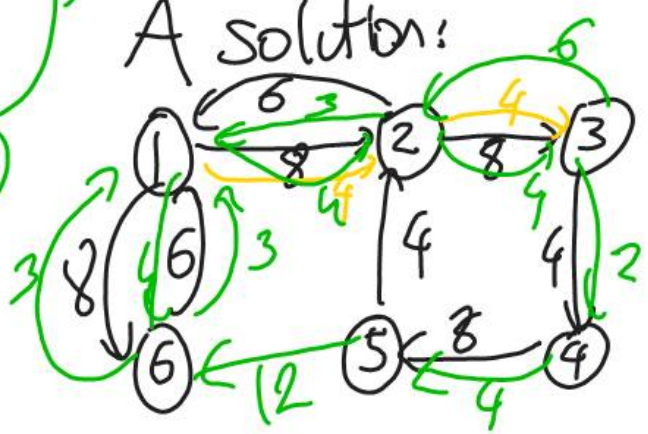
"Snow Removal"

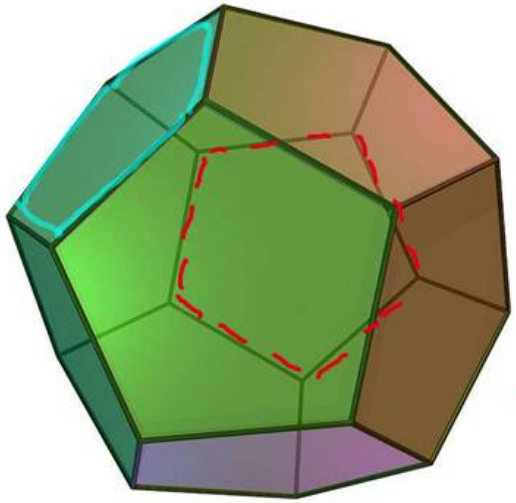


→ minutes to remove the snow near the curb.

It takes half of the duration if the truck just travels w/o sweeping

A solution:



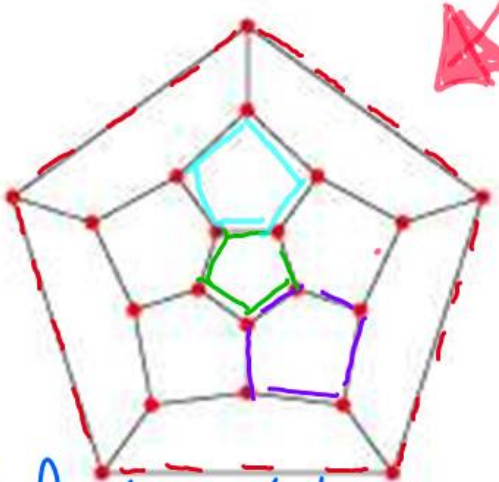


Sir William Rowan Hamilton 1857

Is it possible to visit 20 city network of the globe once & make a tour.

dodecahedron (12 faces) (20 cities)

Hamiltonian Circuit?

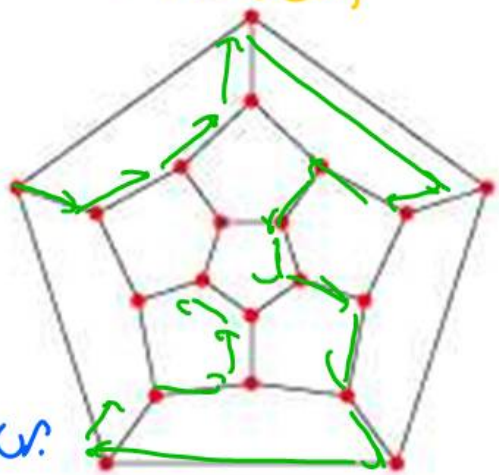


Opt) → The best Hamiltonian Circuit

↕

T.S.P problem

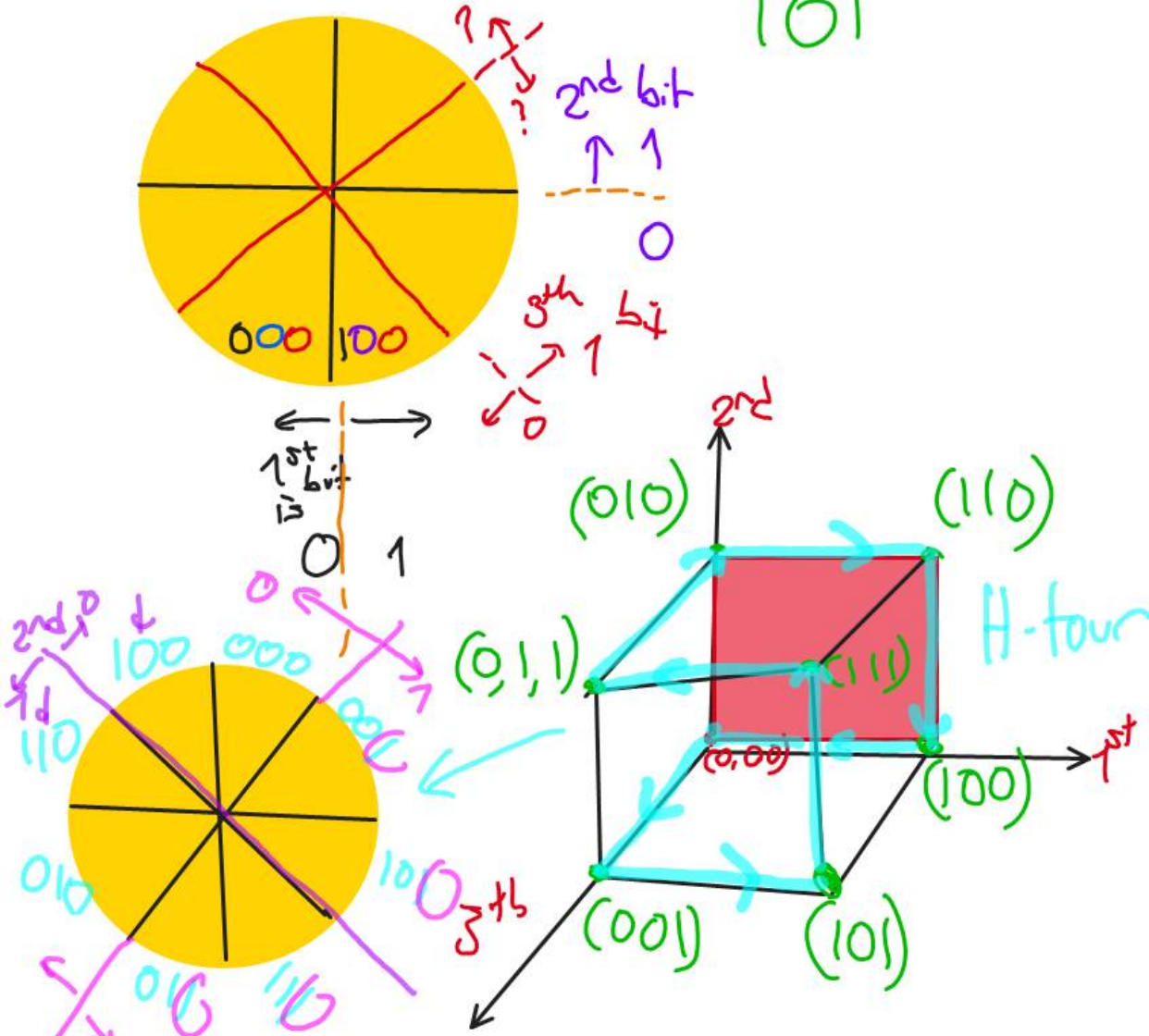
?

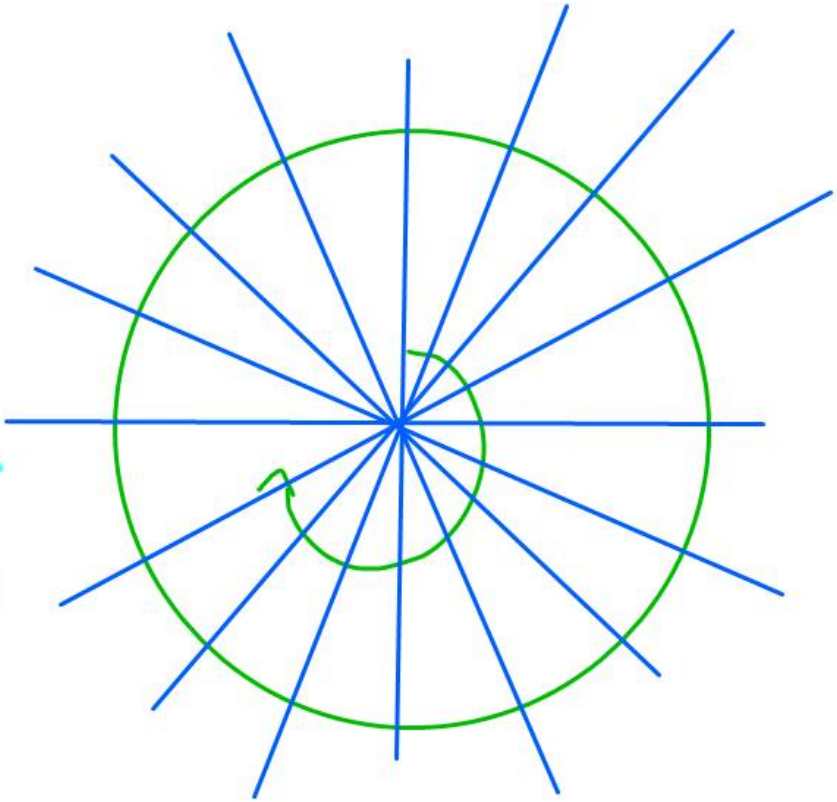


Unfortunately we do not have an easy Necc. & Suff. Cond. for ∃ of a H. tour.

DIGITAL WHEEL OF FORTUNE (Garis-i Felek)

101





TSP ↝ What is the shortest H-tour

↳ The most studied combinatorial opt. prob

Hard to find the optimum sol^n in a reasonable time.

Remedy: Heuristic

- ↳ Construction
- ↳ Improvement

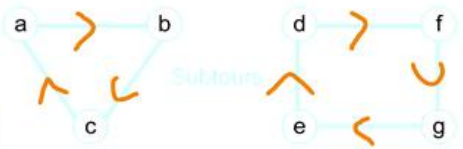
distance : l_1, l_2 ... $x_{ij} = \begin{cases} 1 & \text{if we move to city } j \text{ from } i \\ 0, \infty \end{cases}$

$$\min \sum_{j=1}^{m} \sum_{i=1}^{m} c_{ij} x_{ij}$$

$$\text{s.t.} \sum_{j=1}^{m} x_{ij} = 1 \qquad \text{for } i = 1, \ldots, m$$

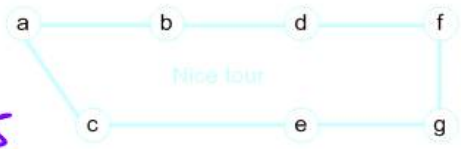
$$\sum_{i=1}^{m} x_{ij} = 1 \qquad \text{for } j = 1, \ldots, m$$

} Assignment



$$\sum_{i \in K} \sum_{j \in K} x_{ij} \leq |K| - 1 \quad \text{for all } K \subset \{1, \ldots, m\}$$

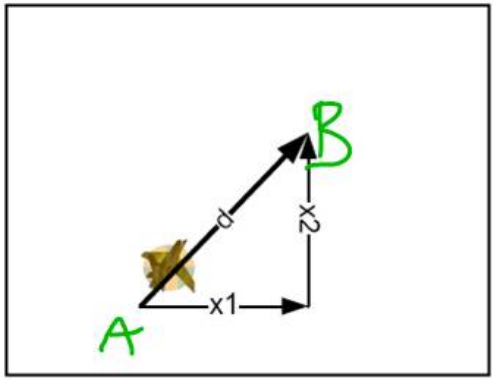
Subtour Elimination Constraints



$2^m, 2, 3, 2^n$

$$x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i, j$$

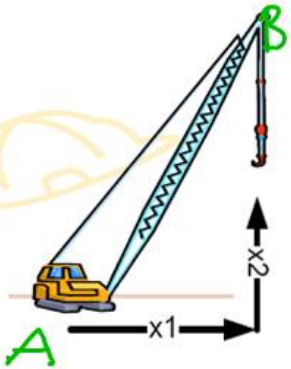
DISTANCE



$d_{AB} = \sqrt{x_1^2 + x_2^2}$

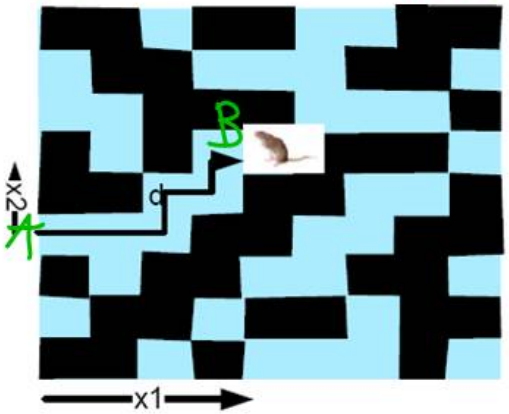
Euclidean Distance

Tchebycheu's Distance

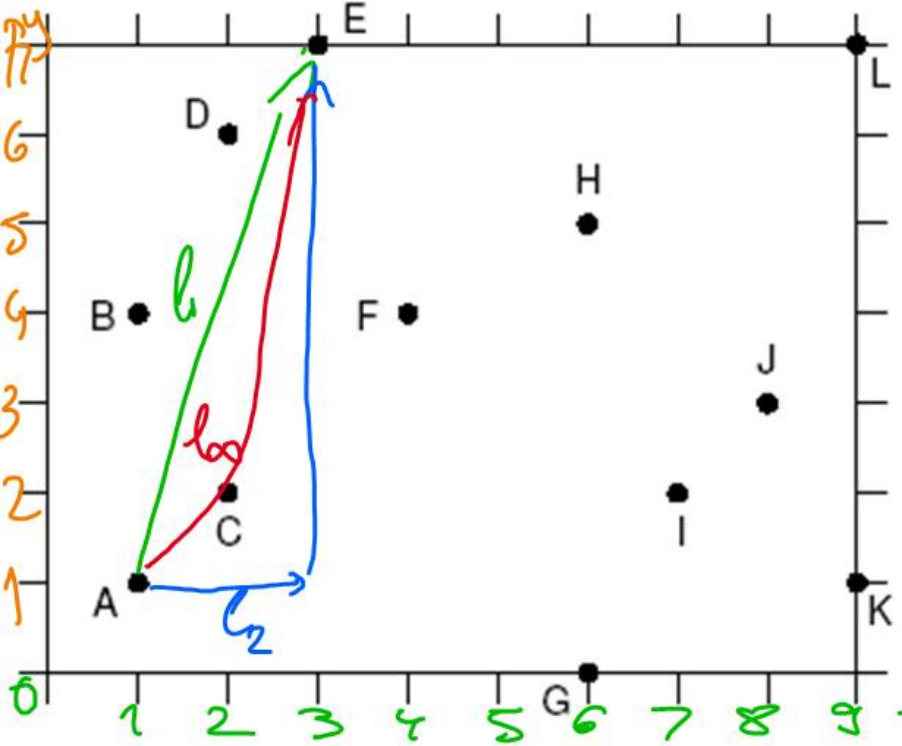


$d_{AB} = \text{Min}\{|x_1|, |x_2|\}$

Manhattan Dist
Rectilinear Dist



$d_{AB} = |x_1| + |x_2|$



$$A=\begin{bmatrix}1\\1\end{bmatrix},\ B=\begin{bmatrix}1\\4\end{bmatrix},\ C=\begin{bmatrix}2\\2\end{bmatrix},\ D=\begin{bmatrix}2\\6\end{bmatrix},\ E=\begin{bmatrix}3\\7\end{bmatrix},\ F=\begin{bmatrix}4\\4\end{bmatrix},\ G=\begin{bmatrix}6\\0\end{bmatrix},\ H=\begin{bmatrix}6\\5\end{bmatrix},\ I=\begin{bmatrix}7\\2\end{bmatrix},\ J=\begin{bmatrix}8\\3\end{bmatrix},\ K=\begin{bmatrix}9\\1\end{bmatrix},\ L=\begin{bmatrix}9\\7\end{bmatrix}.$$

l_∞	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	1	5	6	3	5	5	6	7	8	8
B	3	0	2	2	3	3	5	5	6	7	8	8
C	1	2	0	4	5	2	4	4	5	6	7	7
D	5	2	4	0	1	2	6	4	5	6	7	7
E	6	3	5	1	0	3	7	3	5	5	6	6
F	3	3	2	2	3	0	4	2	3	4	5	5
G	5	5	4	6	7	4	0	5	2	3	3	7
H	5	5	4	4	3	2	5	0	3	2	4	3
I	6	6	5	5	5	3	2	3	0	1	2	5
J	7	7	6	6	5	4	3	2	1	0	2	4
K	8	8	7	7	6	5	3	4	2	2	0	6
L	8	8	7	7	6	5	7	3	5	4	6	0

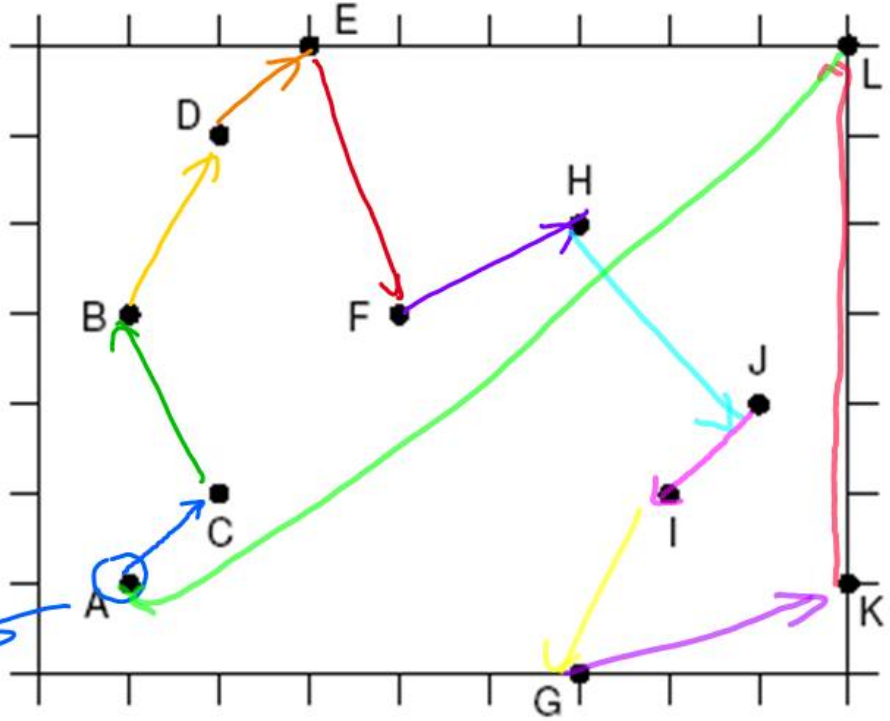
l_2	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	$\sqrt{2}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{18}$	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{37}$	$\sqrt{53}$	8	10
B	3	0	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$	3	$\sqrt{41}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{50}$	$\sqrt{73}$	$\sqrt{73}$
C	$\sqrt{2}$	$\sqrt{5}$	0	4	$\sqrt{26}$	$\sqrt{8}$	$\sqrt{20}$	5	5	$\sqrt{37}$	$\sqrt{50}$	$\sqrt{74}$
D	$\sqrt{26}$	$\sqrt{5}$	4	0	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{40}$	$\sqrt{17}$	$\sqrt{41}$	$\sqrt{45}$	$\sqrt{74}$	$\sqrt{50}$
E	$\sqrt{40}$	$\sqrt{13}$	$\sqrt{26}$	$\sqrt{2}$	0	$\sqrt{10}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{72}$	6
F	$\sqrt{18}$	3	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{10}$	0	$\sqrt{20}$	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{34}$	$\sqrt{34}$
G	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{20}$	$\sqrt{40}$	$\sqrt{58}$	$\sqrt{20}$	0	5	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{10}$	$\sqrt{58}$
H	$\sqrt{41}$	$\sqrt{26}$	5	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{5}$	5	0	$\sqrt{10}$	$\sqrt{8}$	5	$\sqrt{13}$
I	$\sqrt{37}$	$\sqrt{40}$	5	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{13}$	$\sqrt{5}$	$\sqrt{10}$	0	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{29}$
J	$\sqrt{53}$	$\sqrt{50}$	$\sqrt{37}$	$\sqrt{45}$	$\sqrt{41}$	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{8}$	$\sqrt{2}$	0	$\sqrt{5}$	$\sqrt{17}$
K	8	$\sqrt{73}$	$\sqrt{50}$	$\sqrt{74}$	$\sqrt{72}$	$\sqrt{34}$	$\sqrt{10}$	5	$\sqrt{5}$	$\sqrt{5}$	0	6
L	10	$\sqrt{73}$	$\sqrt{74}$	$\sqrt{50}$	6	$\sqrt{34}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{29}$	$\sqrt{17}$	6	$\sqrt{0}$

$$d_{AE}=\{8,\ \sqrt{40},\ 6\}\quad l_1,\ l_2,\ l_\infty$$

l_1	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	2	6	8	6	6	9	7	9	8	14
B	3	0	3	3	5	3	9	6	8	8	11	11
C	2	3	0	4	6	4	6	7	5	7	8	12
D	6	3	4	0	2	4	10	5	9	9	12	8
E	8	5	6	2	0	4	10	5	9	9	12	6
F	6	3	4	4	4	0	6	3	5	5	8	8
G	6	9	6	10	10	6	0	5	3	5	4	10
H	9	6	7	5	5	3	5	0	4	4	7	5
I	7	8	5	9	9	5	3	4	0	2	3	7
J	9	8	7	9	9	5	5	4	2	0	3	5
K	8	11	8	12	12	8	4	7	3	3	0	6
L	14	11	12	8	6	8	10	5	7	5	6	0

EUCLEDIAN

l_2	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	$\sqrt{2}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{18}$	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{37}$	$\sqrt{53}$	8	10
B	3	0	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$	3	$\sqrt{41}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{50}$	$\sqrt{73}$	$\sqrt{73}$
C	$\sqrt{2}$	$\sqrt{5}$	0	4	$\sqrt{26}$	$\sqrt{8}$	$\sqrt{20}$	5	5	$\sqrt{37}$	$\sqrt{50}$	$\sqrt{74}$
D	$\sqrt{26}$	$\sqrt{5}$	4	0	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{40}$	$\sqrt{17}$	$\sqrt{41}$	$\sqrt{45}$	$\sqrt{74}$	$\sqrt{50}$
E	$\sqrt{40}$	$\sqrt{13}$	$\sqrt{26}$	$\sqrt{2}$	0	$\sqrt{10}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{72}$	6
F	$\sqrt{18}$	3	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{10}$	0	$\sqrt{20}$	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{34}$	$\sqrt{34}$
G	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{20}$	$\sqrt{40}$	$\sqrt{58}$	$\sqrt{20}$	0	5	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{10}$	$\sqrt{58}$
H	$\sqrt{41}$	$\sqrt{26}$	5	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{5}$	5	0	$\sqrt{10}$	$\sqrt{8}$	5	$\sqrt{13}$
I	$\sqrt{37}$	$\sqrt{40}$	5	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{13}$	$\sqrt{5}$	$\sqrt{10}$	0	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{29}$
J	$\sqrt{53}$	$\sqrt{50}$	$\sqrt{37}$	$\sqrt{45}$	$\sqrt{41}$	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{8}$	$\sqrt{2}$	0	$\sqrt{5}$	$\sqrt{17}$
K	8	$\sqrt{73}$	$\sqrt{50}$	$\sqrt{74}$	$\sqrt{72}$	$\sqrt{34}$	$\sqrt{10}$	5	$\sqrt{5}$	$\sqrt{5}$	0	6
L	10	$\sqrt{73}$	$\sqrt{74}$	$\sqrt{50}$	6	$\sqrt{34}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{29}$	$\sqrt{17}$	6	$\sqrt{0}$



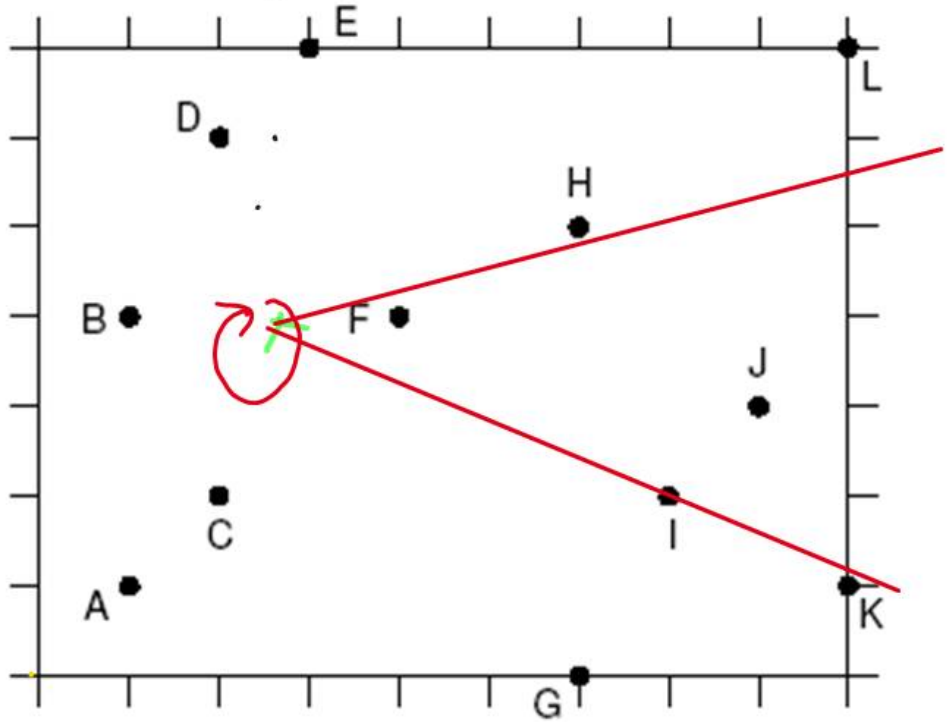
initial location

Nearest Neighbor

Go to the nearest unvisited city

Length = $10+3\sqrt{2}+4\sqrt{5}+2\sqrt{10}+6+\sqrt{8}$

l_2	A	B	C	D	E	F	G	H	I	J	K	L
A	0	3	$\sqrt{2}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{18}$	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{37}$	$\sqrt{53}$	8	10
B	3	0	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$	3	$\sqrt{41}$	$\sqrt{26}$	$\sqrt{40}$	$\sqrt{50}$	$\sqrt{73}$	$\sqrt{73}$
C	$\sqrt{2}$	$\sqrt{5}$	0	4	$\sqrt{26}$	$\sqrt{8}$	$\sqrt{20}$	5	5	$\sqrt{37}$	$\sqrt{50}$	$\sqrt{74}$
D	$\sqrt{26}$	$\sqrt{5}$	4	0	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{40}$	$\sqrt{17}$	$\sqrt{41}$	$\sqrt{45}$	$\sqrt{74}$	$\sqrt{50}$
E	$\sqrt{40}$	$\sqrt{13}$	$\sqrt{26}$	$\sqrt{2}$	0	$\sqrt{10}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{72}$	6
F	$\sqrt{18}$	3	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{10}$	0	$\sqrt{20}$	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{34}$	$\sqrt{34}$
G	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{20}$	$\sqrt{40}$	$\sqrt{58}$	$\sqrt{20}$	0	5	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{10}$	$\sqrt{58}$
H	$\sqrt{41}$	$\sqrt{26}$	5	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{5}$	5	0	$\sqrt{10}$	$\sqrt{8}$	5	$\sqrt{13}$
I	$\sqrt{37}$	$\sqrt{40}$	5	$\sqrt{41}$	$\sqrt{41}$	$\sqrt{13}$	$\sqrt{5}$	$\sqrt{10}$	0	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{29}$
J	$\sqrt{53}$	$\sqrt{50}$	$\sqrt{37}$	$\sqrt{45}$	$\sqrt{41}$	$\sqrt{17}$	$\sqrt{13}$	$\sqrt{8}$	$\sqrt{2}$	0	$\sqrt{5}$	$\sqrt{17}$
K	8	$\sqrt{73}$	$\sqrt{50}$	$\sqrt{74}$	$\sqrt{72}$	$\sqrt{34}$	$\sqrt{10}$	5	$\sqrt{5}$	$\sqrt{5}$	0	6
L	10	$\sqrt{73}$	$\sqrt{74}$	$\sqrt{50}$	6	$\sqrt{34}$	$\sqrt{58}$	$\sqrt{13}$	$\sqrt{29}$	$\sqrt{17}$	6	$\sqrt{0}$



SWEEP

Choose a reference point

As we rotate a line pivoted at that point

Order the cities as we encounter by rotating say clockwise

D → E → L → H → F → J → I → K → G → A → B → D

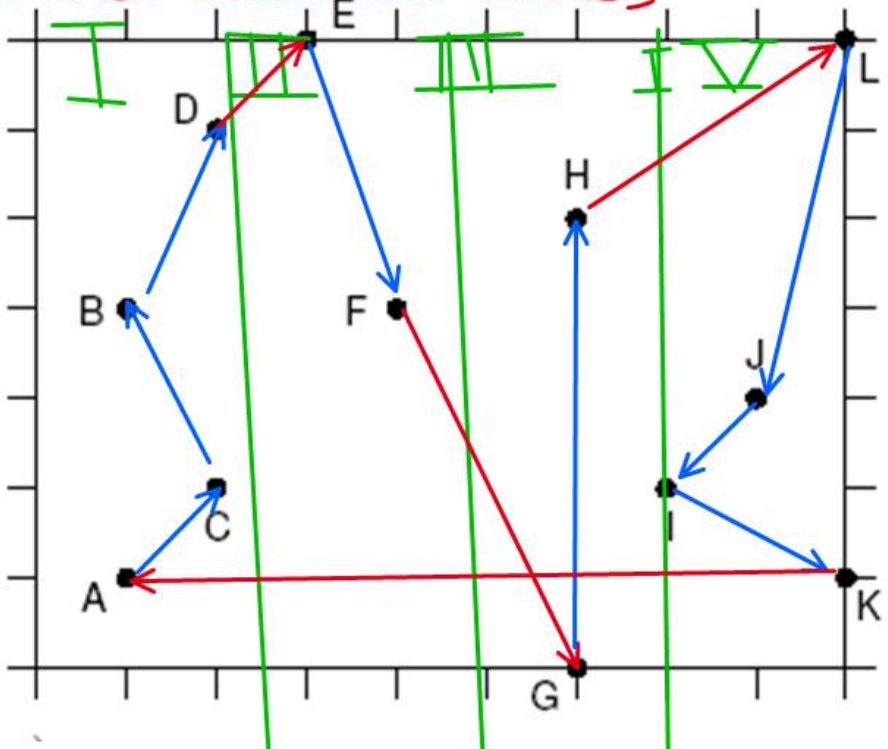
Length

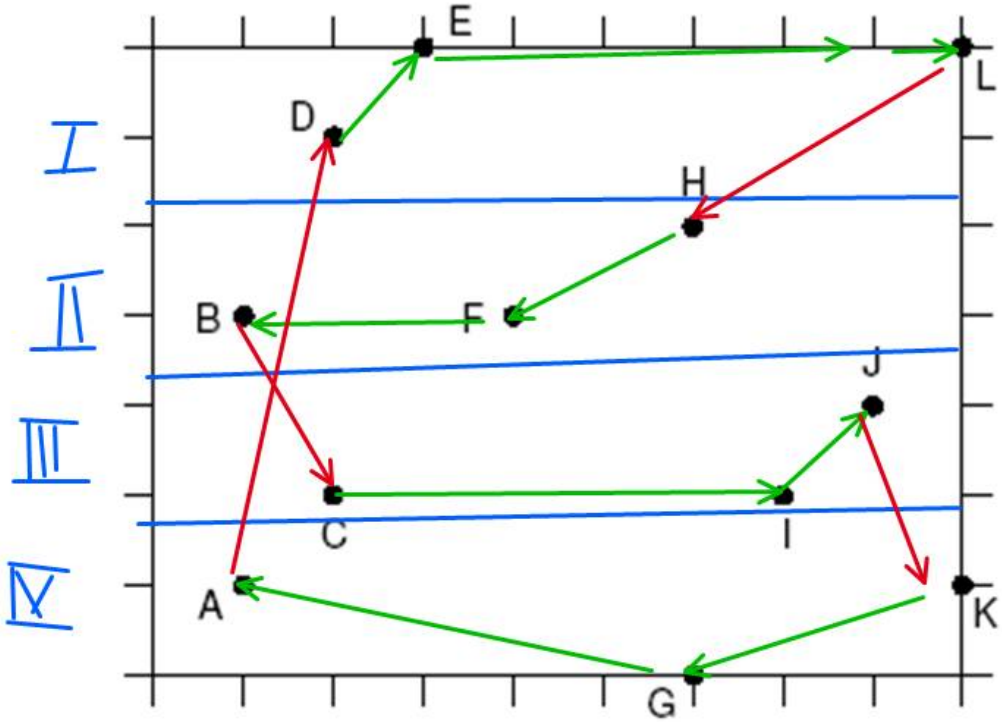
$3+6+\sqrt{13}+\sqrt{17}+\sqrt{20}$

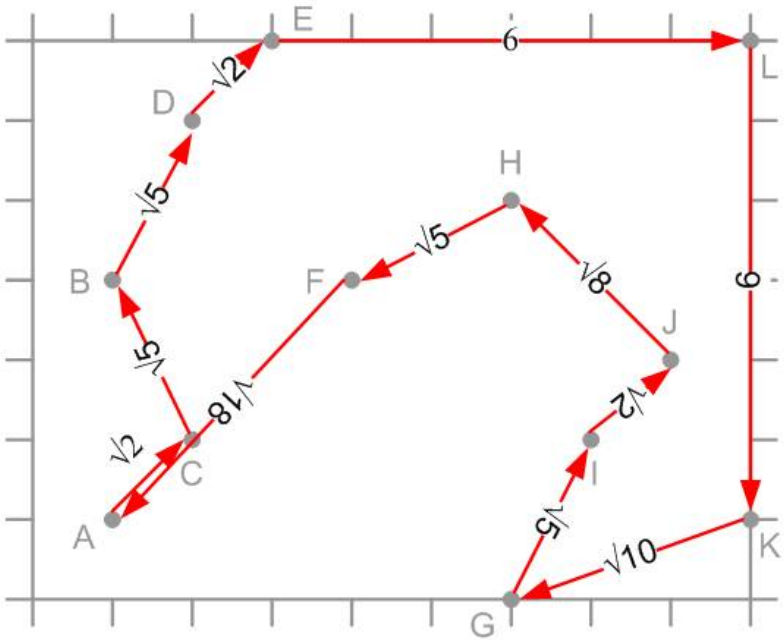
BAND Heur

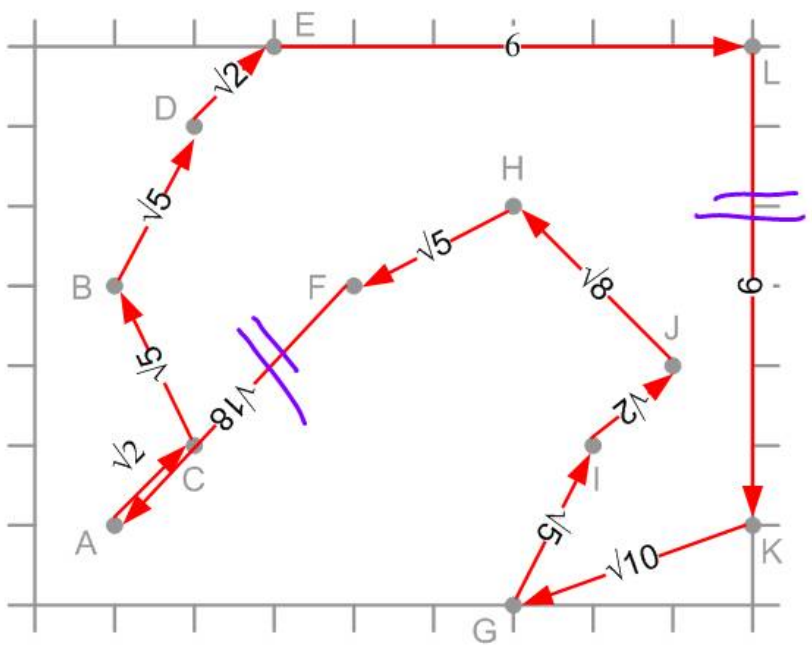
Divide the region into horizontal vertical strips (bands) of even number

Connect the cities in odd even bands in ↗ ✕ Values Connect the consecutive bands






E
6
L
D
√2
H
√5
B
F
√5
√8
9
J
√5
√18
√2
√2
C
I
A
√5
√10
K
G


E
6
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√2
C
I
A
√5
√10
K
G