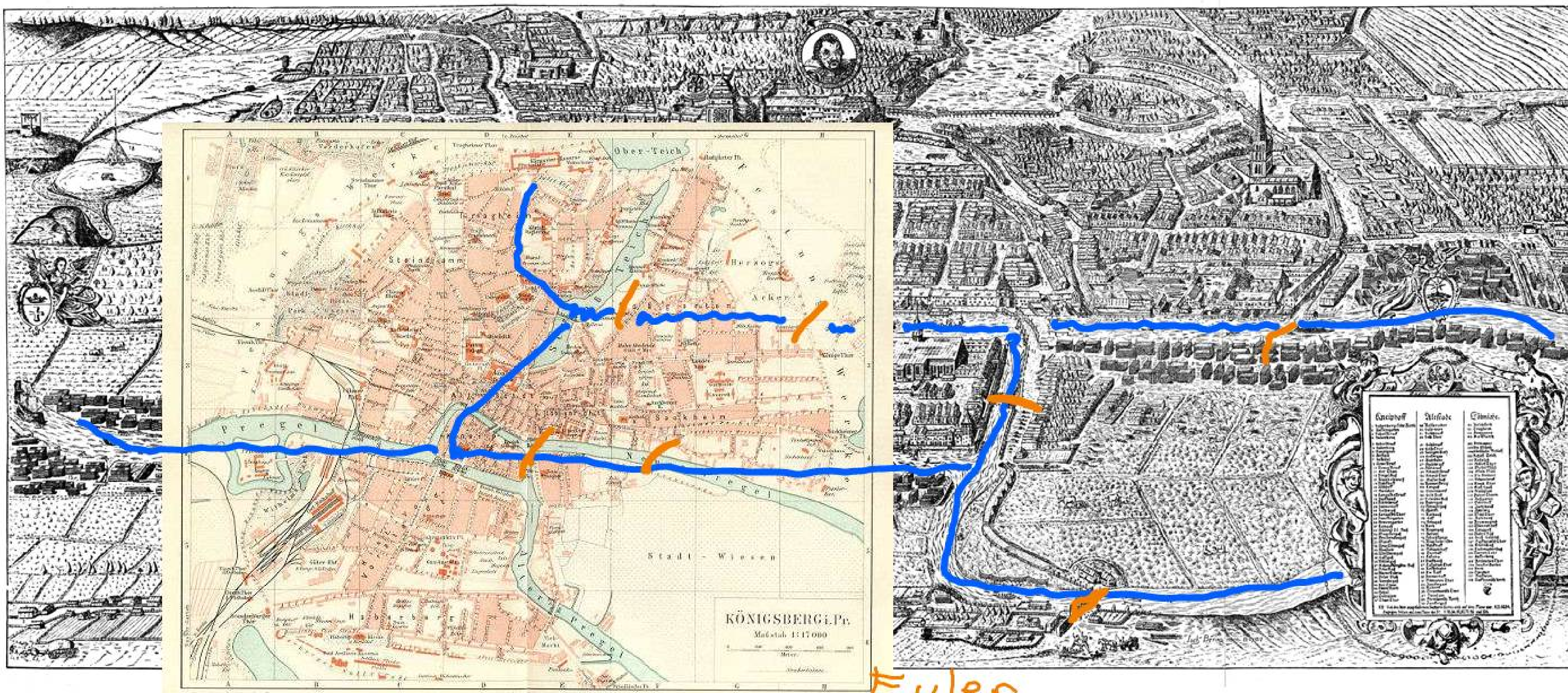
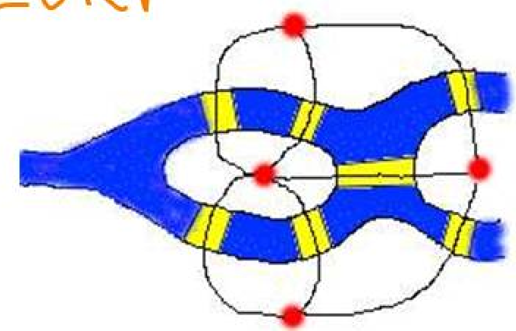
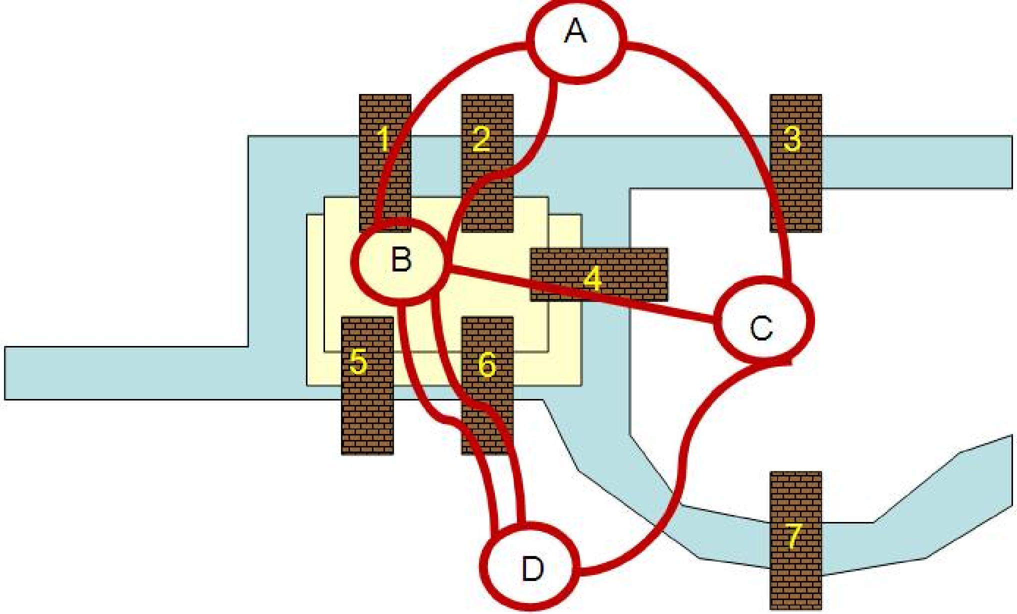
Gedenkblatt zur sechshundert jährigen Jubelfeier der Königlichen Haupt und Residenz-Stadt Königsberg in Preußen.



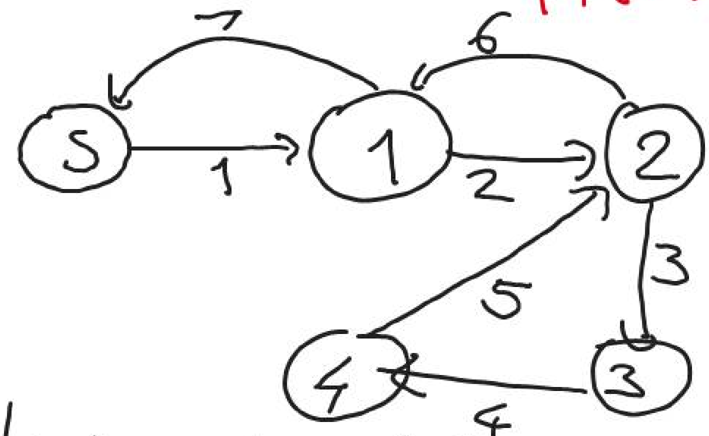

Euler

7 Bridges of Konigsberg Prussia






# EULERIAN CIRCUITS
# TOURS
# PATHS

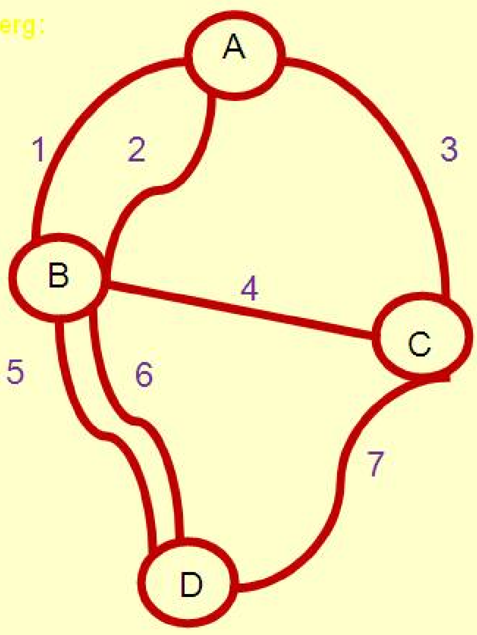



Eulerian circuit (tour) in $G=(V,E)$ is a tour (start & stop at a certain vertex) such that we visit every edge once.

The Bridges of Koenigsberg:
Euler 1736

People wondered whether it is possible to take a walk, end up where you started from, and cross each bridge in Koenigsberg exactly once

**NO!**

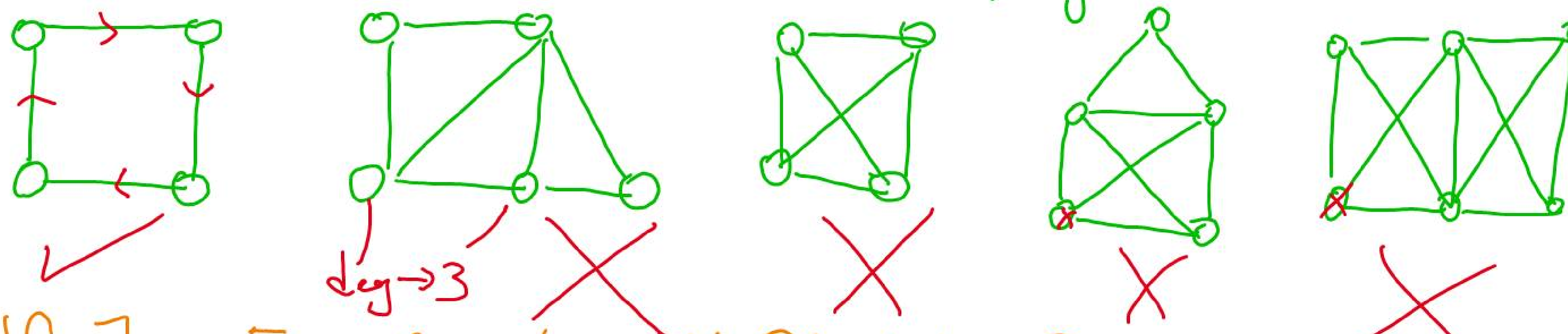



$\exists$ a neccessary & sufficient condition for the existence of a EULERIAN circuit

$\exists$ Eulerian Circuit $\Longleftrightarrow$ $\deg(v) = \text{even} \ \forall v \in V$ ALL!

in a $G=(V,E)$

↳ # of edges incident to $v$




If $\exists$ a E-circuit HOW to DETECT it?

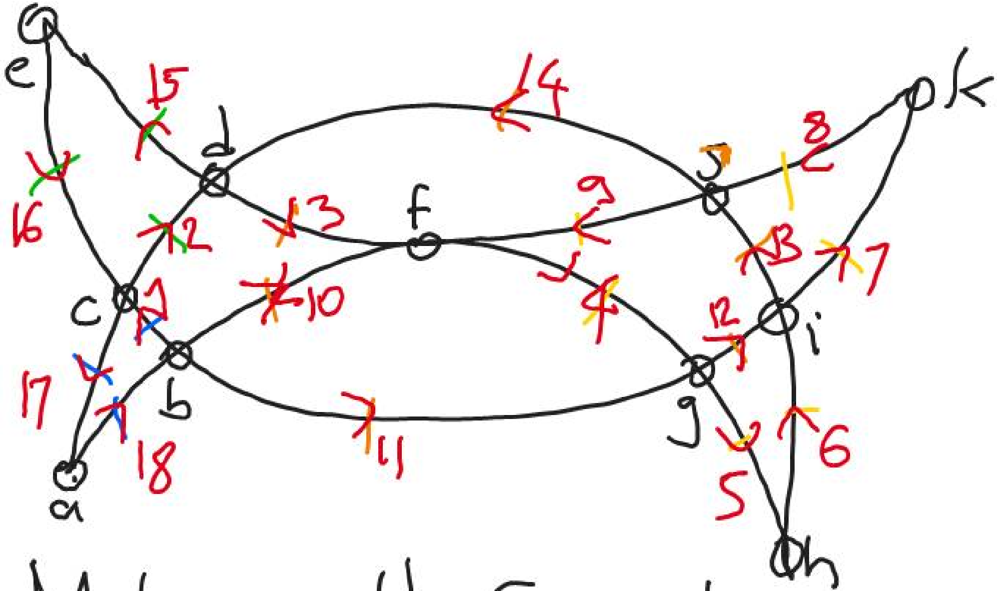
Algo

S1. Identify cycles: $C_1, \dots, C_k$

S2. If there is no edge left $\xrightarrow[\text{Eulerian}]{\text{graph is}}$

ow There is no E-circuit

Start at any vertex
Take a cycle containing that vertex. Traverse this cycle until you hit another unvisited cycle, traverse the new cycle
Continue in LIFO fashion




C1: a → b → c

C2: d → e → c

C3: d → f → b → g → i → j

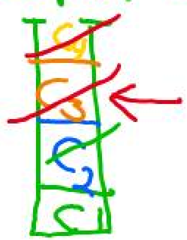
C4: k → j → f → g → h → i

Mohammed's Scimeter
[Pala]

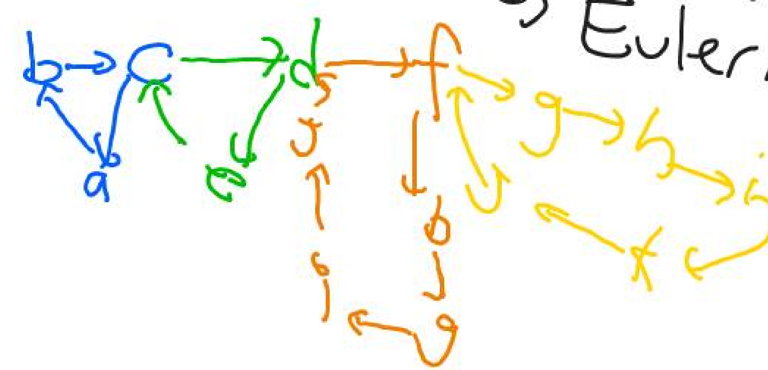
$\deg(v) = \text{even} \ \forall v \in V$

This graph contains an E-circuit

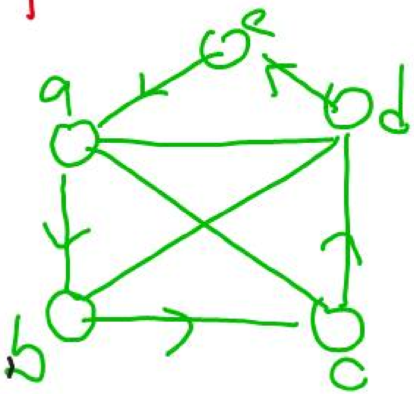
Start with b & $C_1$




No edges left ⤳ Eulerian

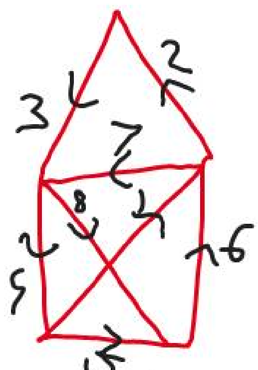


If there are ONLY two vertices of odd degree, we can find an Eulerian PATH → A path containing all the edges!




Can you draw this graph w/o taking your pen off the paper?

→ YES if $\exists$ a E-path OR E-circuit




a → b → c → d → e → a : cycle

left → path

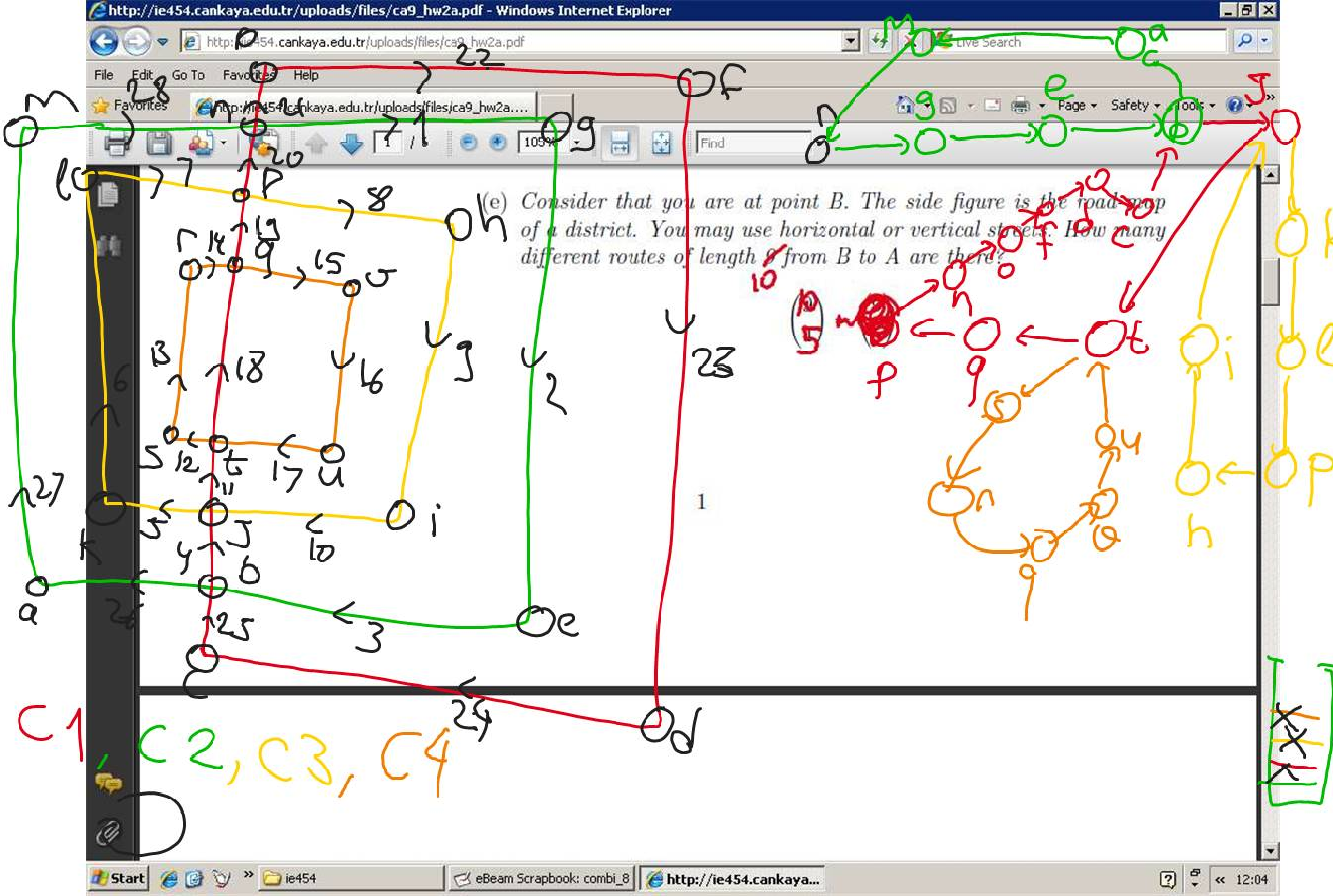
b → d → a → c

Start with the path:

b → d → e → a → b → c → d

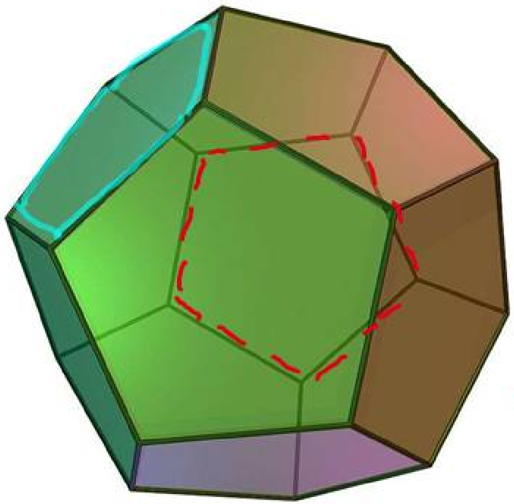
d — a

c ← a



(e) *Consider that you are at point B. The side figure is the road map of a district. You may use horizontal or vertical streets. How many different routes of length ~~8~~ 10 from B to A are there?*

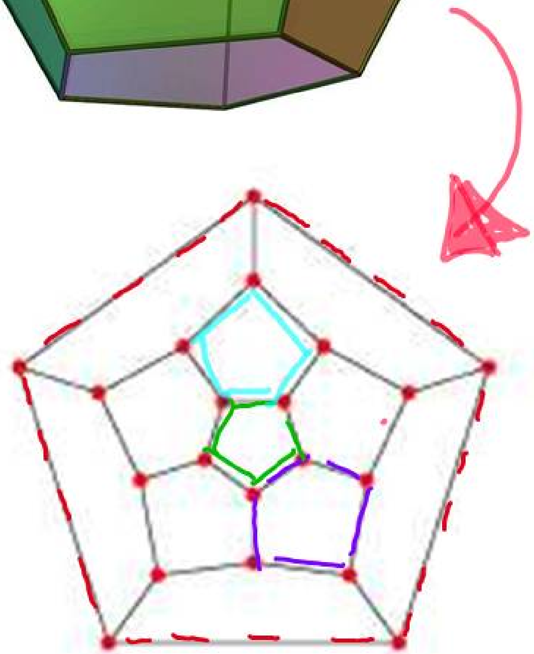
1

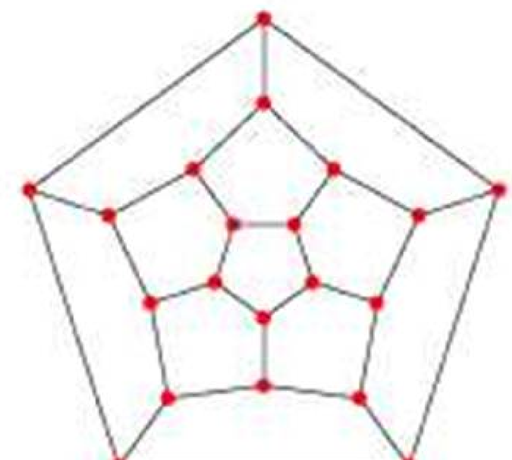


Sir William Rowan Hamilton 1857

dodecahedron (12 faces)

(20 cities)





$$\left[1+x+x^2+x^3\right]^5$$

$$g(x) = 1+x+x^2+x^3+\cancel{x^4+x^5}+\cdots$$

$$x^4 g(x) = \qquad\qquad \cancel{x^4+x^5+x^6}+\cdots$$

$$-$$

$$\left[1-x^4\right]g(x) = 1+x+x^2+x^3 = \left[1-x^4\right]\left[1+x+x^2+x^3+x^4+\cdots\right]$$

$$\left[1+x+x^2+x^3\right]^5 = \left[\quad\right]^5\left[\quad\right]^5$$