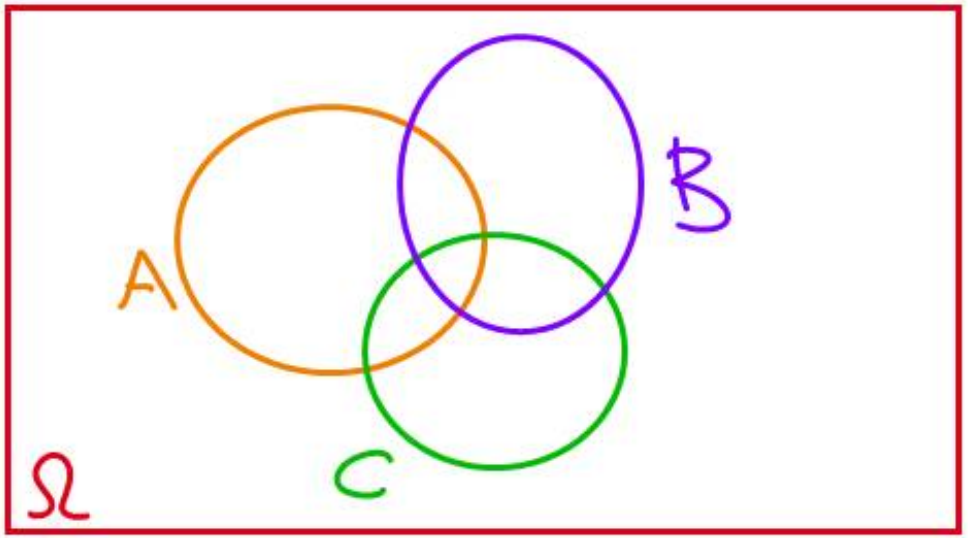
INCLUSION & EXCLUSION



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$n(\overline{A \cup B \cup C}) = n(\Omega) - n(A \cup B \cup C)$$
$$= n(\Omega) - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)$$

Example

What is the numbers between 1 and 1000 that are (i) divisible by 2 or 5
(ii) non-divisible by 2 or 5 or 11

A B C

A = integers between 1 & 1000 that are divisible by 2

$n(A) = \left\lfloor \frac{1000}{2} \right\rfloor = 500$

$n(B) = \left\lfloor \frac{1000}{5} \right\rfloor = 200$

$n(C) = \left\lfloor \frac{1000}{11} \right\rfloor = 90$

B = integers "

C = integers "

$\lfloor 3.14 \rfloor = 3$ floor function: take the integer part

$\lceil 3.14 \rceil = 4$ ceiling " : take the next integer

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 500 + 200 - 100 = 600$

$n(A \cap B) = \left\lfloor \frac{1000}{2(5)} \right\rfloor$

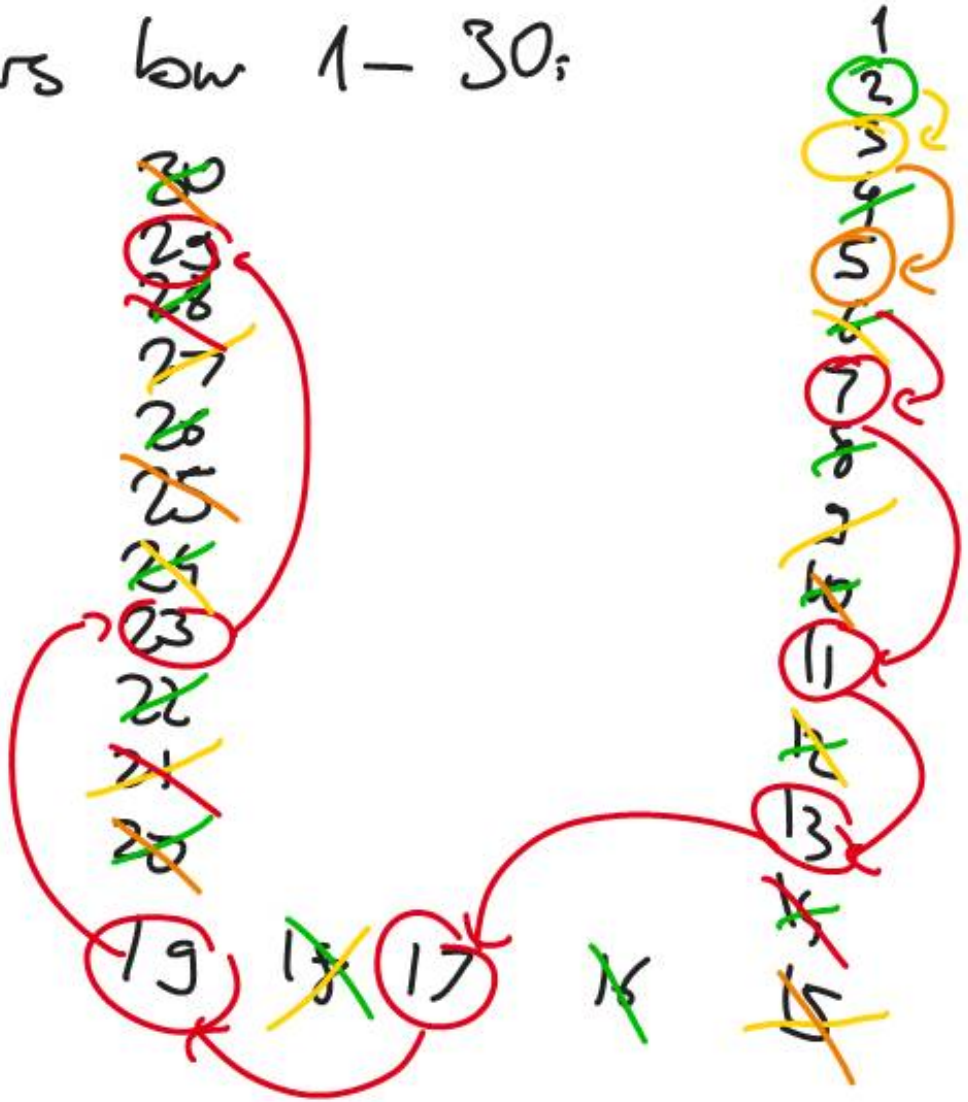
(ii) $n(\overline{A \cup B \cup C}) = n(\Omega) - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)$

$= 1000 - 500 - 200 - 90 + 100 + 45 + 18 - 9$

$= 364$

The Sieve of Eratosthenes

The prime numbers low 1 – 30:

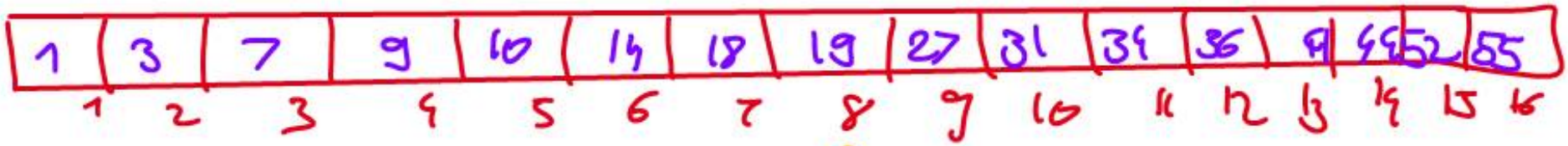


DIVIDE & CONQUER

Binary Search of a key integer in a sorted array

→ Yes ← location

No



Key = 10 left=1 right=16

middle = $\lfloor \frac{1+16}{2} \rfloor = 8$

10 ? 19 NO

10 < 19 ✓ right←7

middle = $\lfloor \frac{1+7}{2} \rfloor = 4$

10 ? 9 NO

10 > 9 left←5

middle = $\lfloor \frac{5+7}{2} \rfloor = 6$

10 ? 16 NO

10 < 14 right←5

middle = $\lfloor \frac{5+5}{2} \rfloor = 5$

bingo

At each iteration we cut the array in halves!

Suppose $f(n)$ is a function denoting the # of operations needed to solve a problem of size n.

If we can divide at each iteration a (sub)problem on hand into a many subproblems of size b reducing if it takes c operations to combine the solutions.

$$f(n) = a\, f\left(\left\lfloor \frac{n}{b} \right\rfloor\right) + c$$

$$= a\left[a \cdot f\left(\left\lfloor \frac{n}{b^2} \right\rfloor\right) + c\right] + c$$

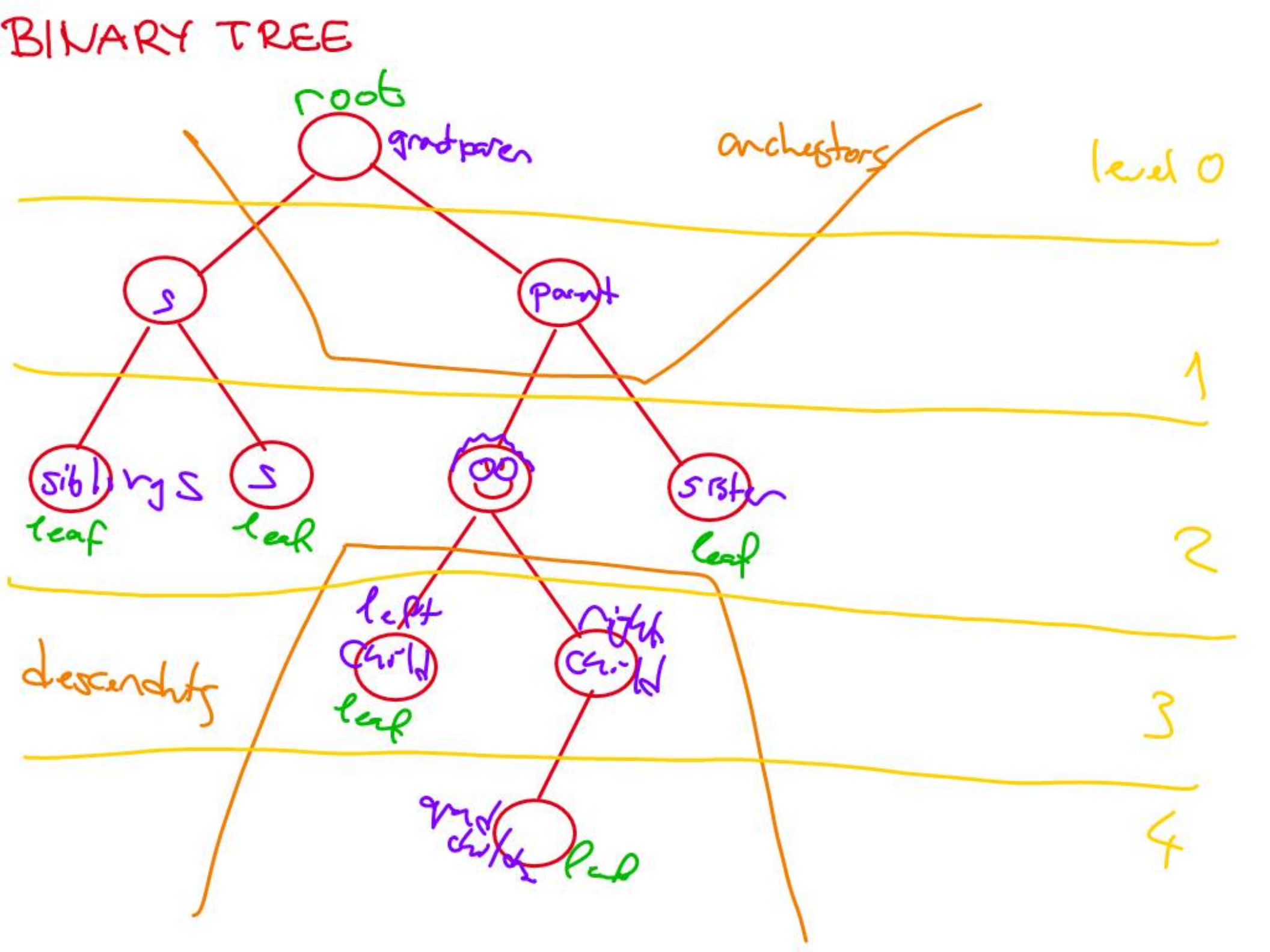
$$\left\lfloor \frac{n}{b^{(k)}} \right\rfloor = 1 \quad \Rightarrow k = \log_b n$$

$$\vdots$$

$$f(n) = a^k f\left(\left\lfloor \frac{n}{b^k} \right\rfloor\right) + c\left[1 + a + a^2 + \cdots + a^{k-1}\right] \cong a^{\log_b n}$$

$$\vdots$$

$$f(n) = a\, f\left(\left\lfloor \frac{n}{b^k} \right\rfloor\right) + c$$


BINARY TREE
root
grandparent
ancestors
level 0
parent
1
2
3
4
descendants
leaf
leaf
leaf
leaf
leaf

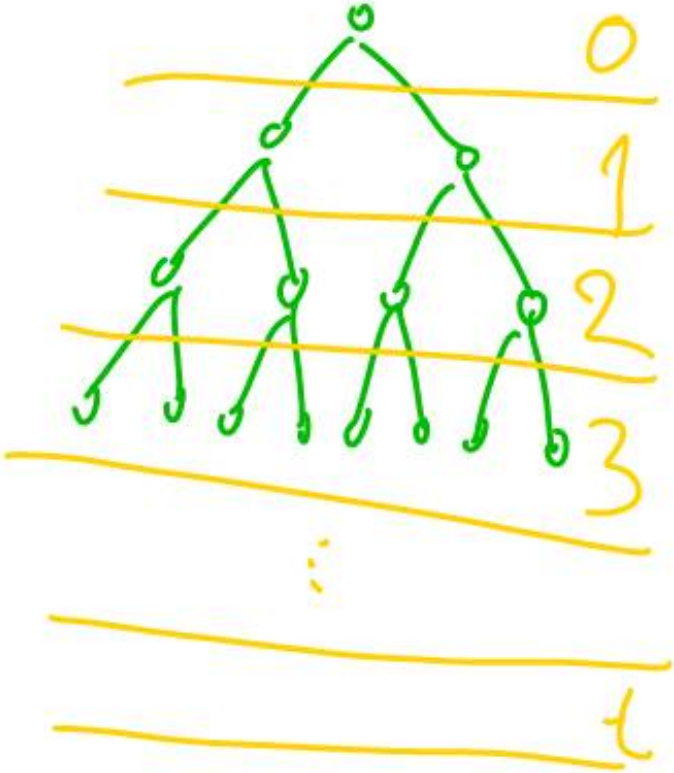
A binary tree is called COMPLETE if every parent has exactly two children

Question In a complete binary tree, how many nodes are there?

(i) at level t 2^t

(ii) in a complete BT of depth t

$2^0 + 2^1 + \cdots + 2^t = 2^{t+1} - 1$



Traversals on BTs

(i) In order

(ii) pre order

(iii) post order

lpr

plr

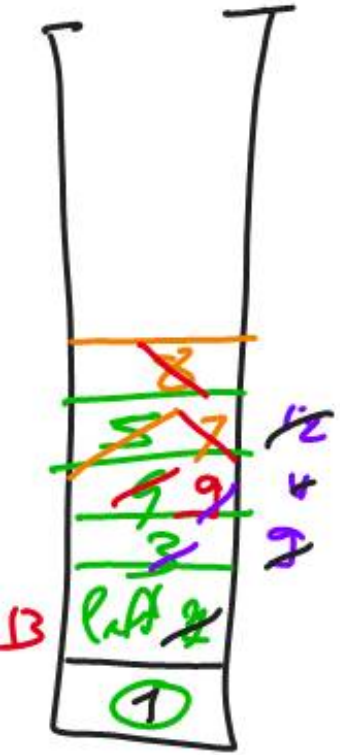
lrp

applied recursively

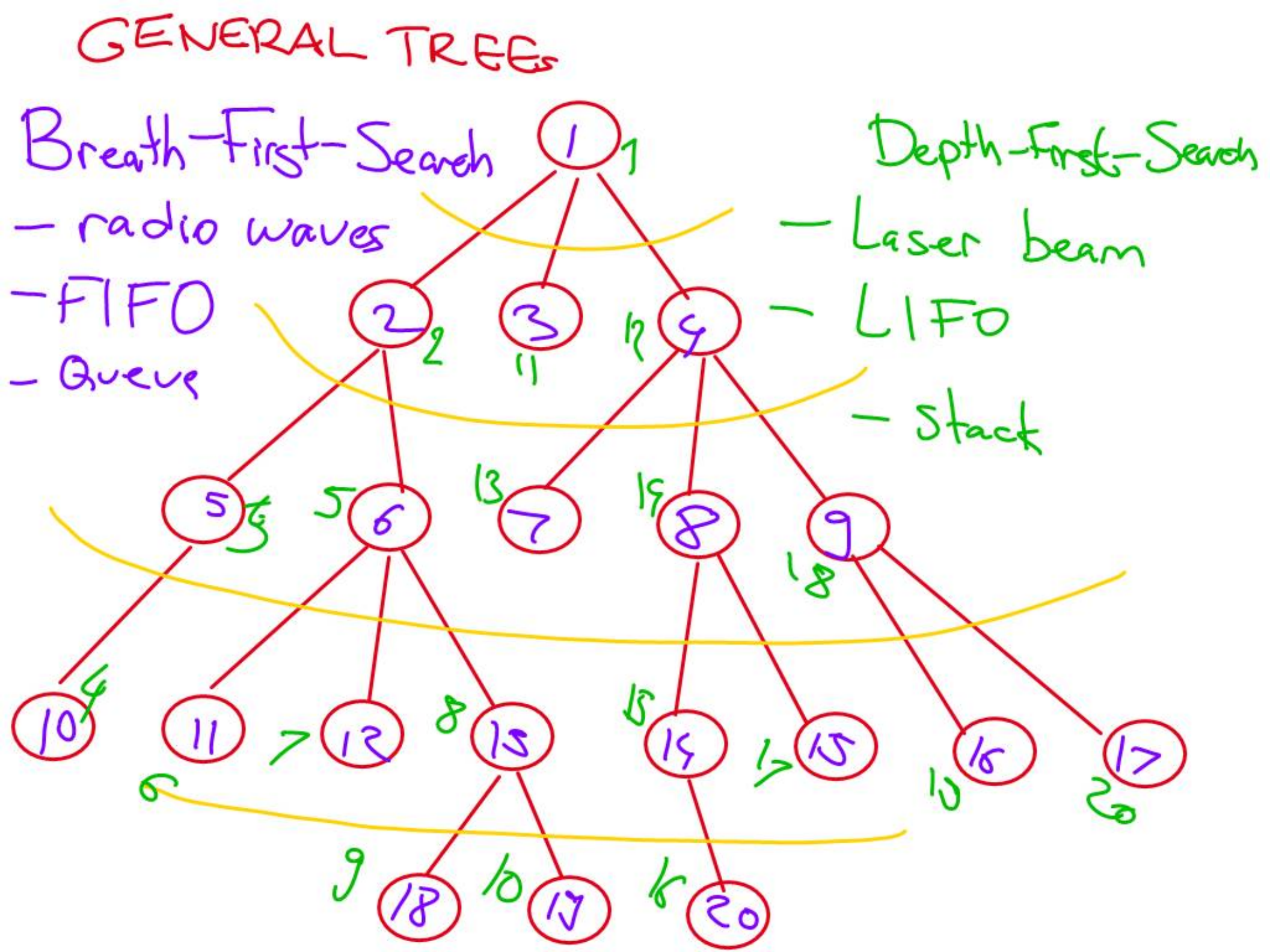
preorder: plr

In order: lpr

Post order: lrp

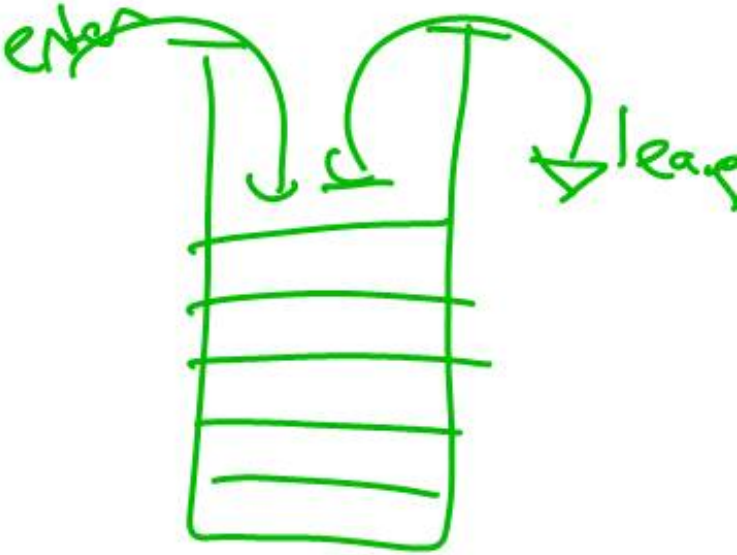


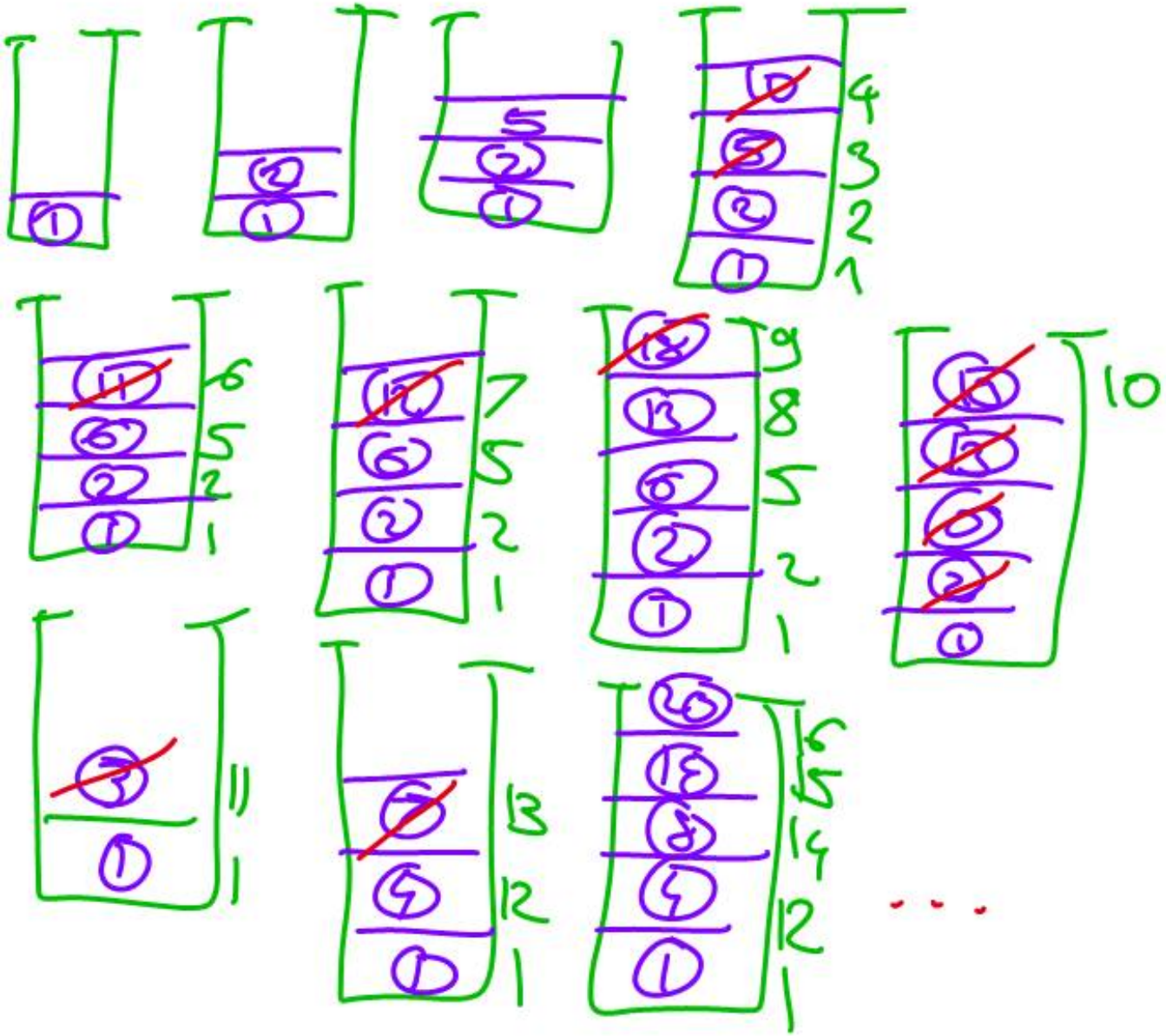
GENERAL TREES



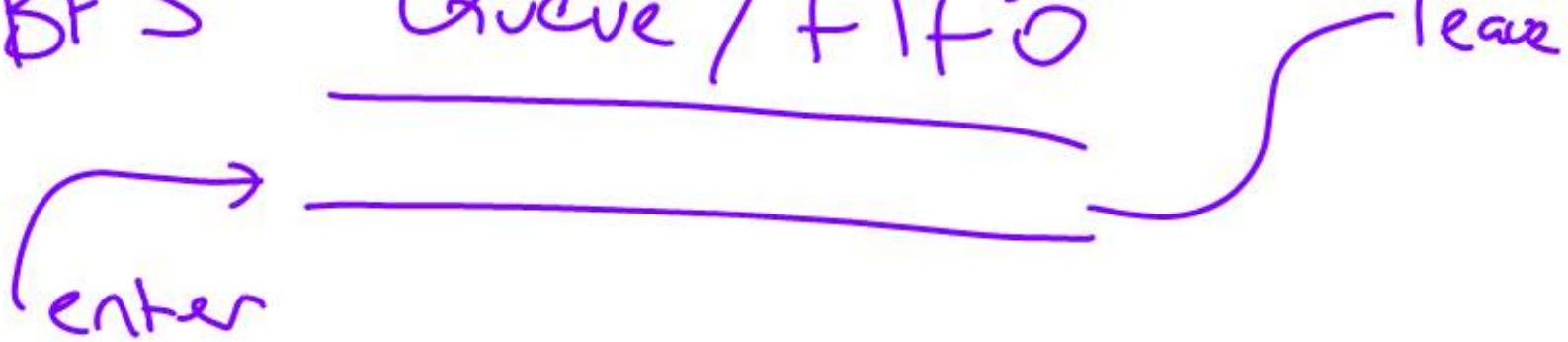
DFS

UFO – Stack



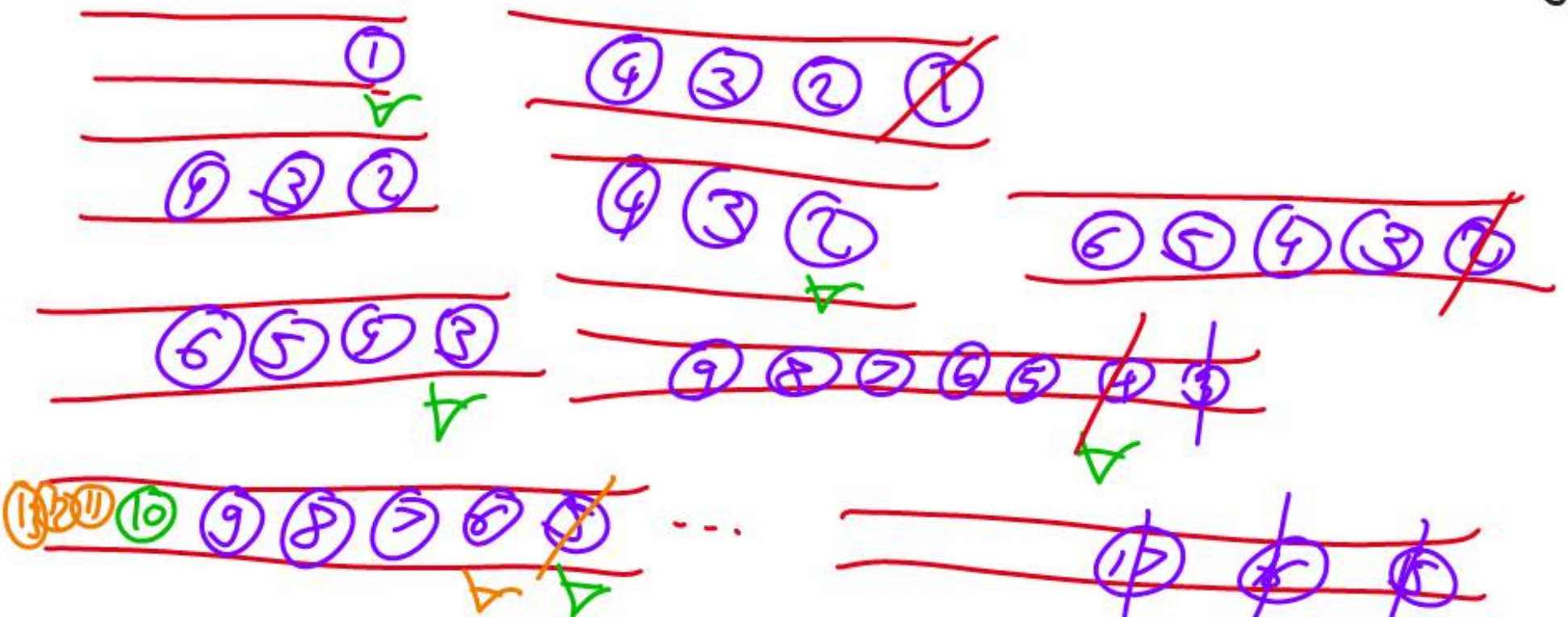


BFS Queue / FIFO



Examine the top (rightmost, front) element in the queue

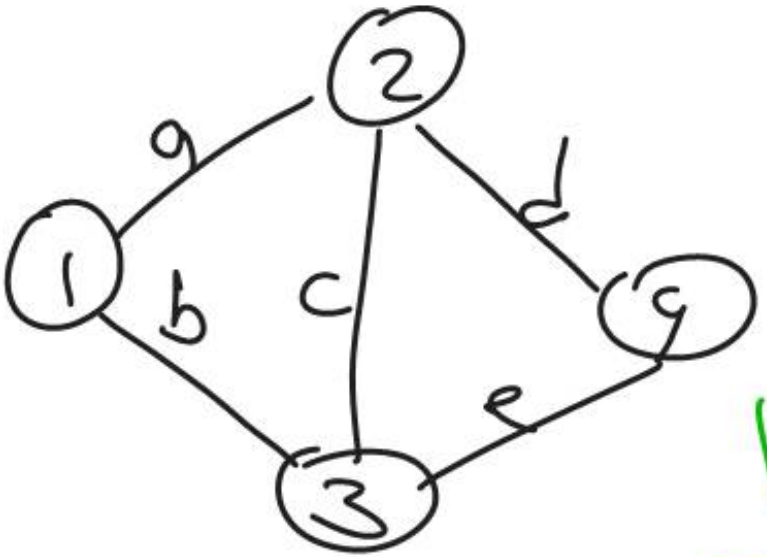
- Add all its children into the queue — from left to right
- Delete the top



TRAVERSALS of GRAPHS

$G = (V, E)$

- vertex set (node)
- edge set



$V = \{1, 2, 3, 4\}$

$E = \{a, b, c, d, e\}$

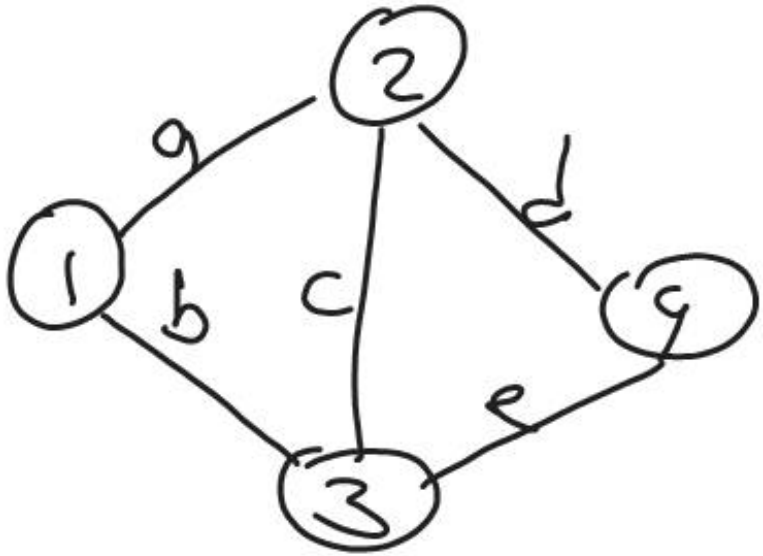
$e \in E,\ e = \{i, j\}$ → end points of e

$d \in E$

$d = \{2, 4\}$

Node-Node Connectivity Matrix

	1	2	3	4
1	0	1	1	0
2	1	0	1	1
3	1	1	0	1
4	0	1	1	0



Symmetric

Node-edge Incidence Matrix

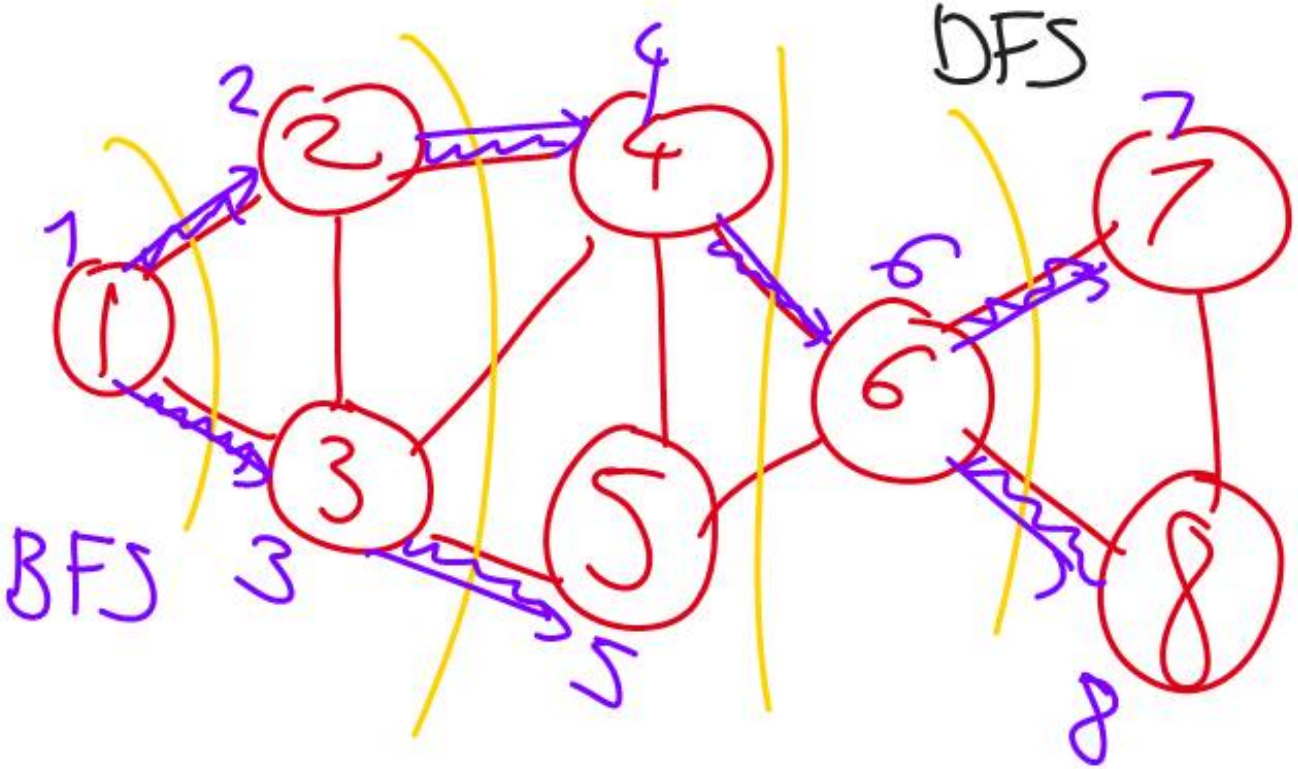
	a	b	c	d	e
1	1	1	0	0	0
2	1	0	1	1	0
3	0	1	1	0	1
4	0	0	0	1	1

Structure

TRAVERSE GRAPHS

- BFS — Start at any vertex, r : root
- DFS — Apply BFS to all unvisited nodes

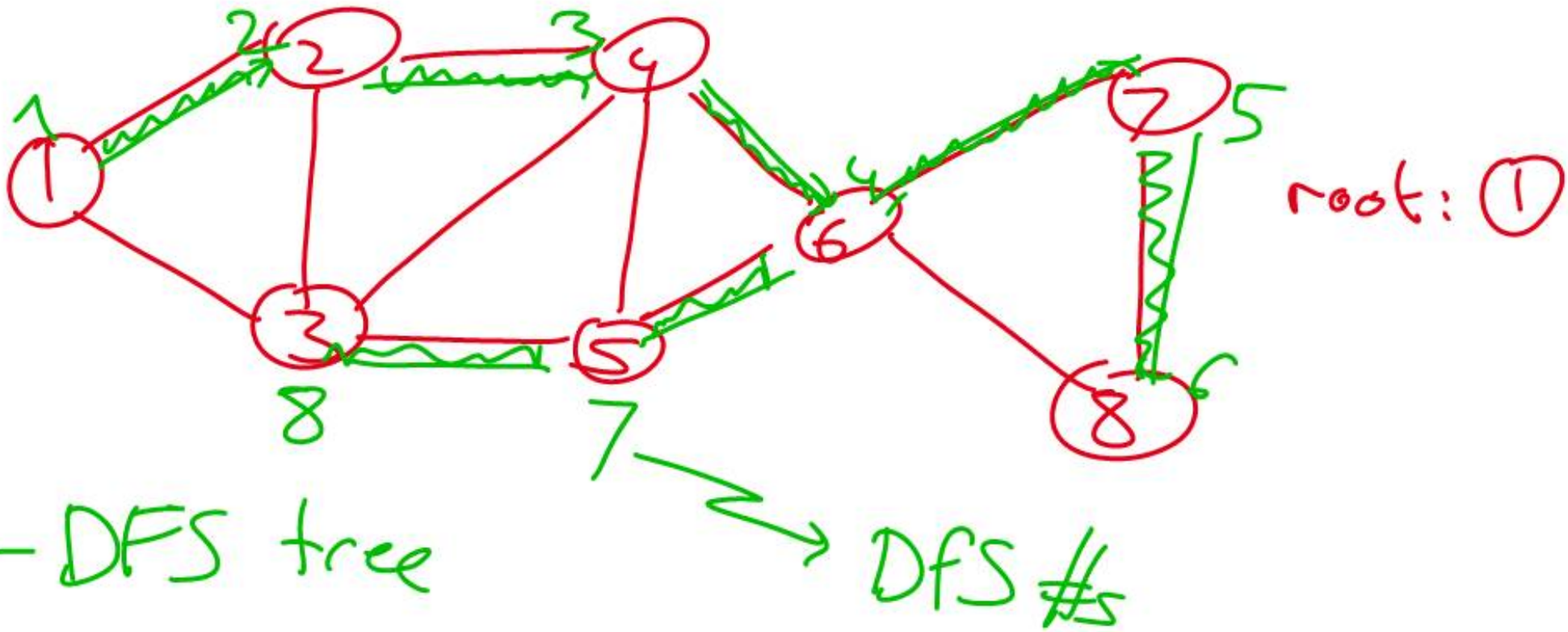
DFS



root : 1

BFS tree

BFS #s.



— DFS tree

8 7 → DFS #s