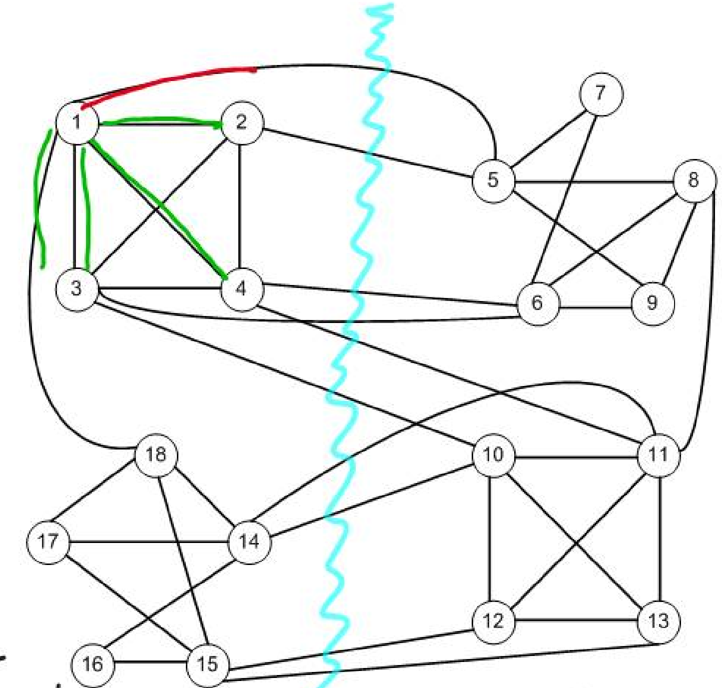
IMPROVEMENT (for BISECTION)

a) Kernighan & Lin (K&L)

↳ swap (interchange) of two nodes across the cut.



$\gamma(V_1)$ v External : in the cut $\delta(V_1;V_2)$

Internal : not in the cut

$I(v) = \sum_{e\ \text{internal}} w_e$

$E(v) = \sum_{e:\ \text{external}} w_e$

↳ Increase in the cut value if we move node v to the other side

$Gain(v) = I(v) - E(v)$

V_1 10 V_2

$I(1) = 4$ $E(1) = 1$ $Gain(1) = 3$

edge weights = same

A) $w_e = 1\ \forall e \in E$

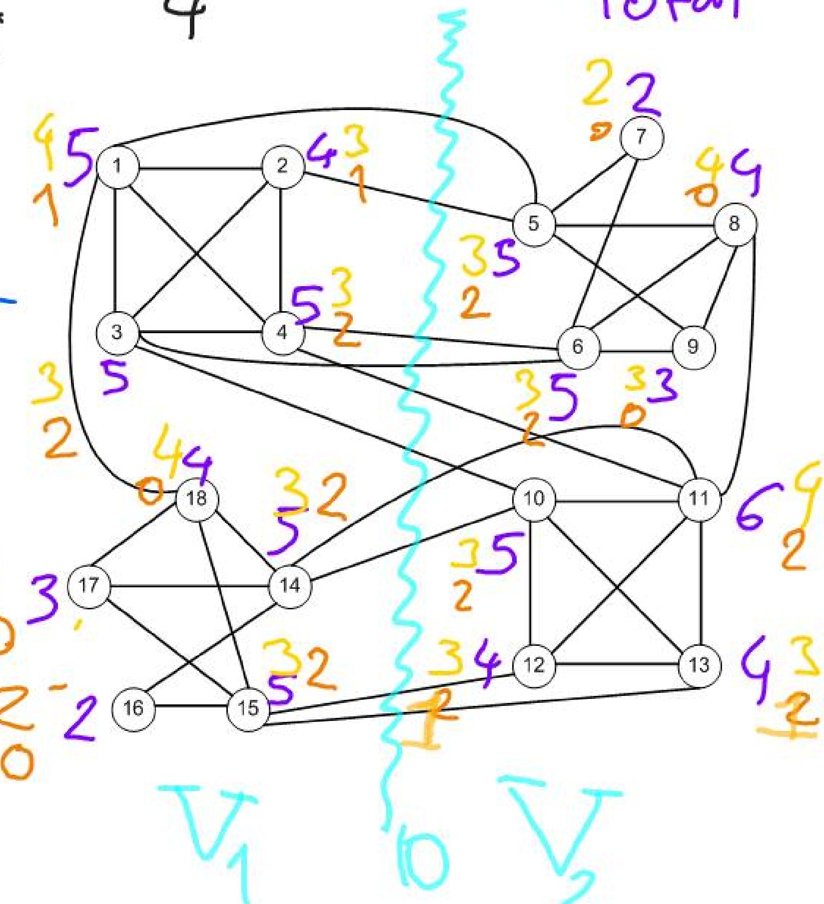
$Gain(s;t) = Gain(s) + Gain(t) - 2w_{st}^{*}$

If ∃ edge b/w s & t

GAIN(i,j)

GAIN		1	1	2	4	3	1	2	2	2
GAIN	NODE	NODE 5	6	7	8	9	10	11	12	13
3	1	2	4	5	7	6	4	5	5	5
2	2	1	3	4	6	5	3	4	4	4
1	3	2	0	3	5	4	0	3		
1	4	2	0	3	5	4	2	1		
1	14	2	2	3	5	4	0	1		
1	15	2	2	3	5	4				
2	16	3	3	4						
3	17	4	4							
4	18	5	5							

External
+ Internal
Total



For Min Cut   For Max Cut

HW #4 – Apply the below procedure for the example prob– for (i) max-cut and, (ii) min-cut

S1. Calculate Gain(i;j)'s

S2. If the minimum/maximum value in the gain matrix is negative/positive then make the interchange
freeze the nodes
go Back to S1.
else STOP

Improvement

(ii) Fiduccia–Mathyses (F–M)

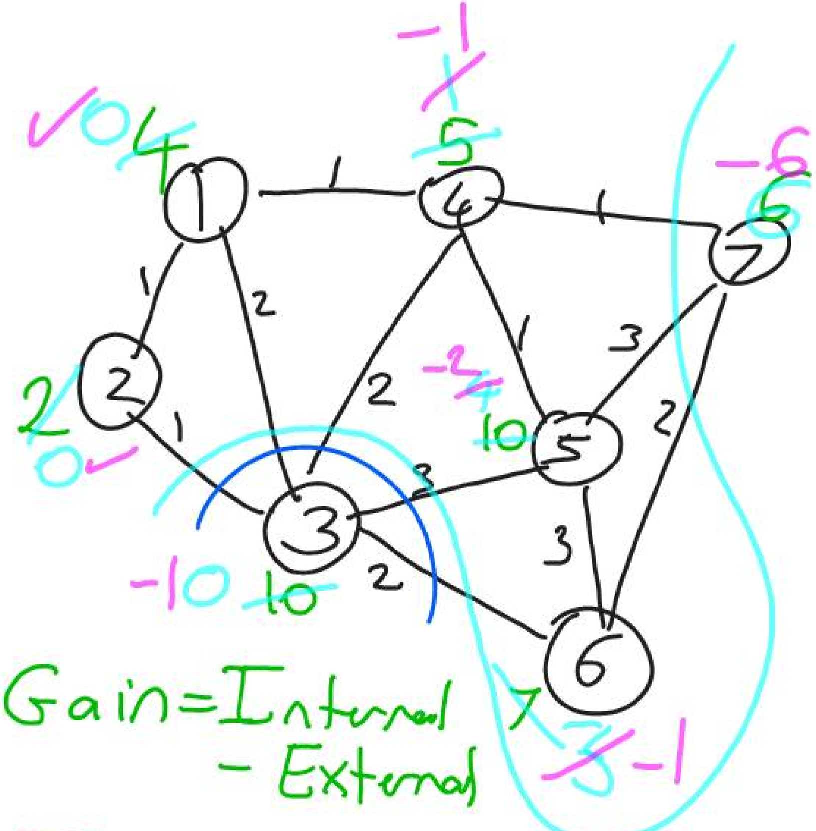
"One node" at a time   accross the CUT

S1. Calculate the gain(i) = Internal(i) − Externd(i)
$\forall i$

S2. Choose the min/max gain value, say α

If $\alpha > 0 \Rightarrow$ make the move and freeze the node
@ STOP

$\alpha < 0 \Rightarrow$ make the move & freeze the node
@ STOP



Gain = Internal − External

$S = \emptyset$ Max $\delta(S)$

Step 1 $\alpha = 10$ Tie : 3 , 5

Tie breaking Rule : Lexicographical order

$S = \{3\}$ cutsize 10

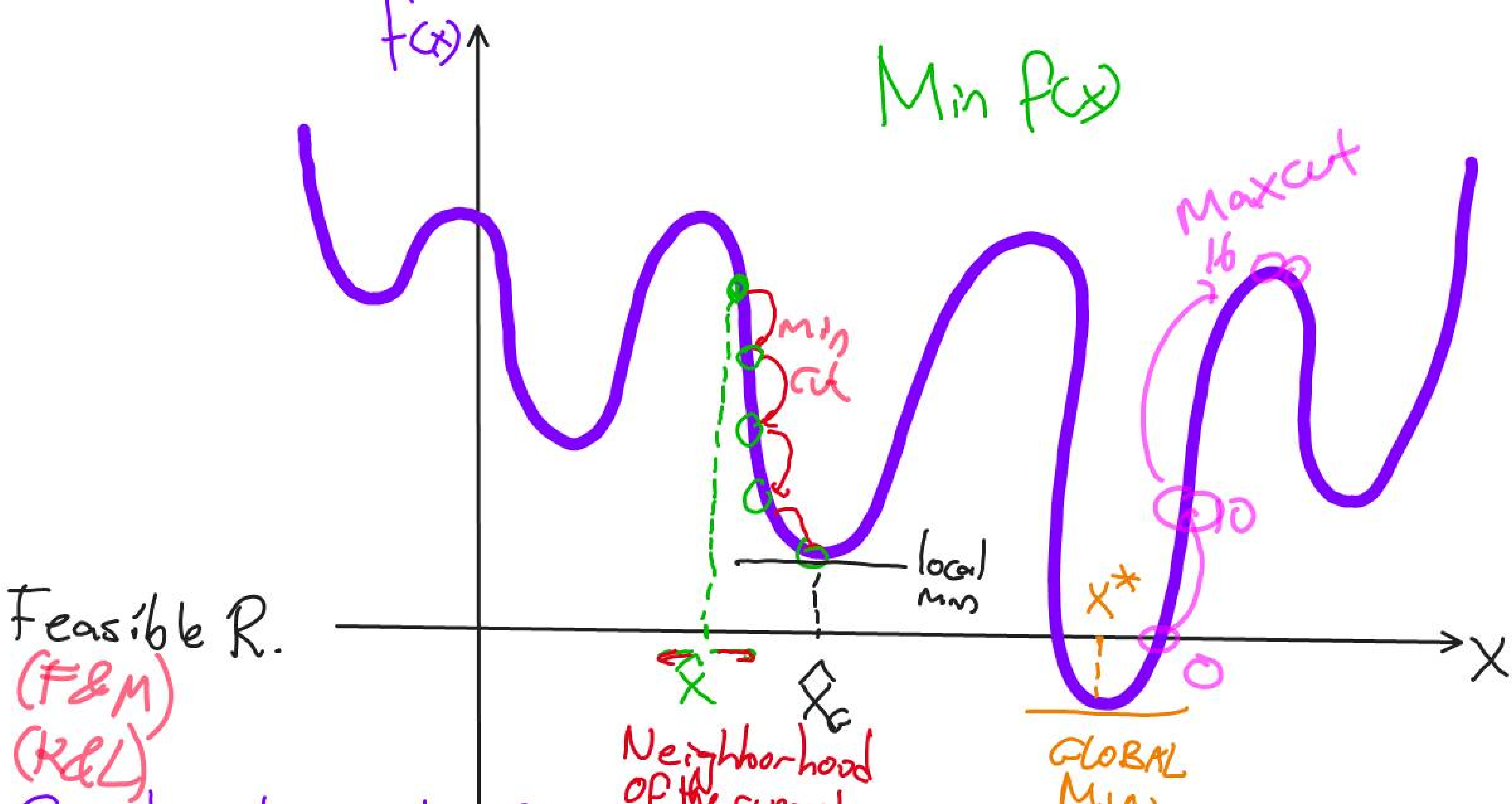
Step 2 $\alpha = 6 \rightarrow 7$

$S = \{3, 7\}$ cutsize 16

Step 3 $\alpha = 0$ STOP

(If we are NOT interested in alternative opt. solutions)

F-M → max-cut problem.



Greedy stops at $\hat{x}_G$

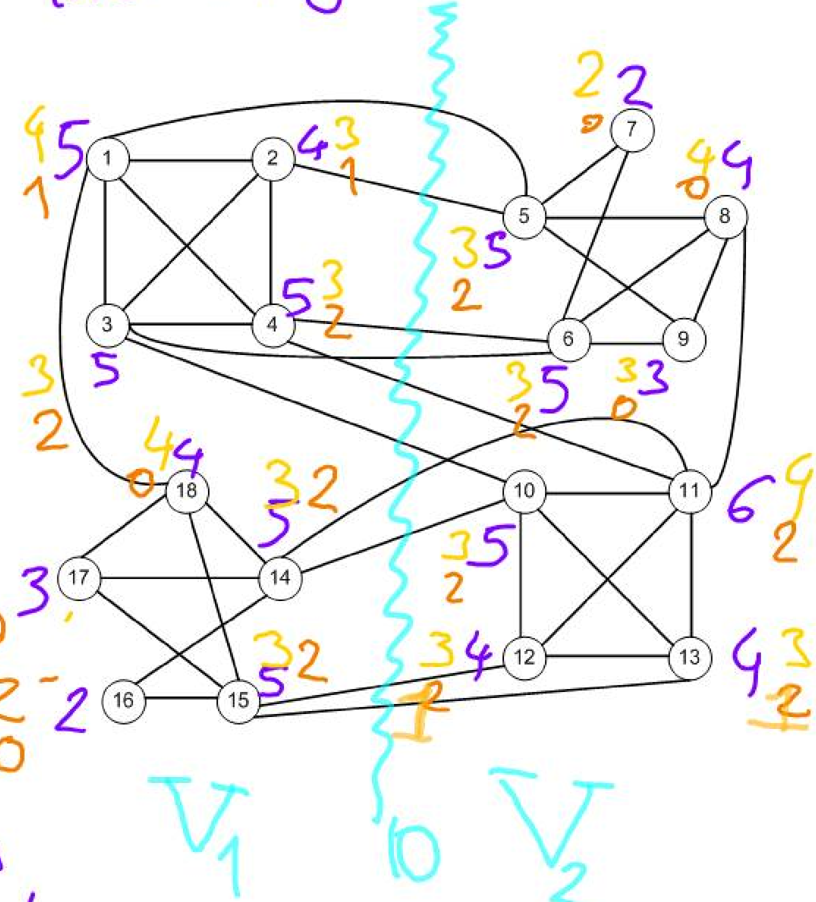
$\boxed{\text{Non greedy}}$ tries to climb up-hill with the hope of getting a better solution.

The GREEDY (myopic) heuristic stops if there is no improvement

Consider the min_cut problem below: The min. gain is 0.

ITERATION 1	
MIN	0

				5	6	7	8	9	10	11	12	13
			Internal	3	3	2	4	3	3	4	3	3
			External	2	2	0	0	0	2	2	1	1
			Gain	1	1	2	4	3	1	2	2	2
Internal	**External**	**Gain**	**NODE**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
4	1	3	**1**	2	4	5	7	6	4	5	5	5
3	1	2	**2**	1	3	4	6	5	3	4	4	4
3	2	1	**3**	2	0	3	5	4	0	3	3	3
3	2	1	**4**	2	0	3	5	4	2	1	3	3
3	2	1	**14**	2	2	3	5	4	0	1	3	3
3	2	1	**15**	2	2	3	5	4	2	3	1	1
2	0	2	**16**	3	3	4	6	5	3	4	4	4
3	0	3	**17**	4	4	5	7	6	4	5	5	5
4	0	4	**18**	5	5	6	8	7	5	6	6	6



We may obtain a better solution if we let zero (even negative) gained moves

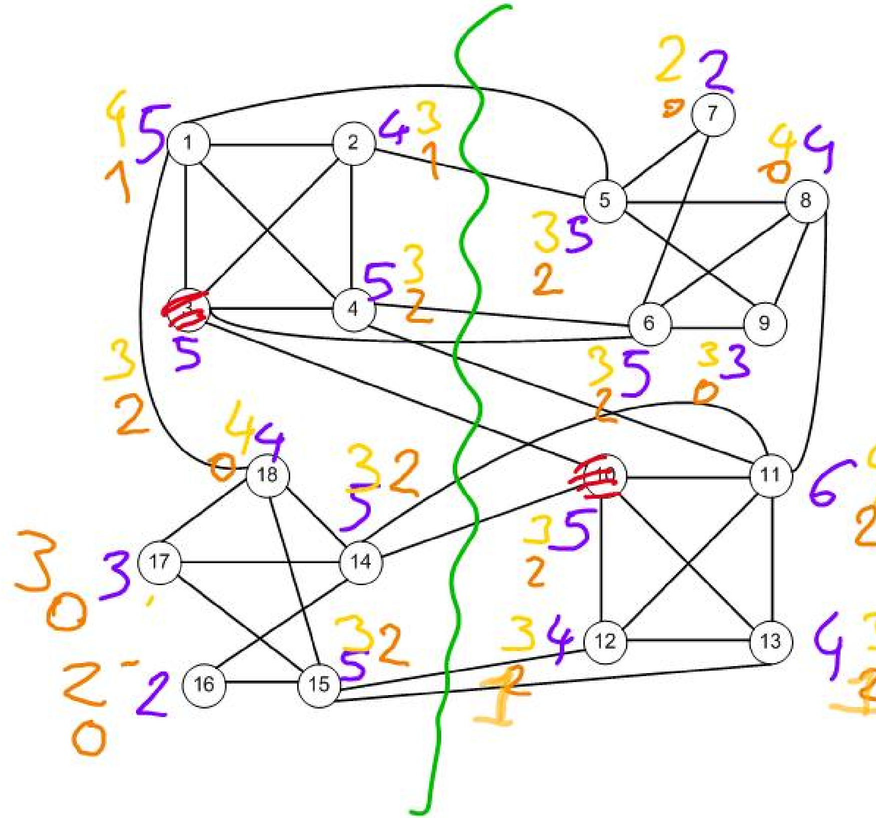
→ NON-GREEDY

Nongreedy with threshold 2 for MIN-CUT PROBLEM

ITERATION 1

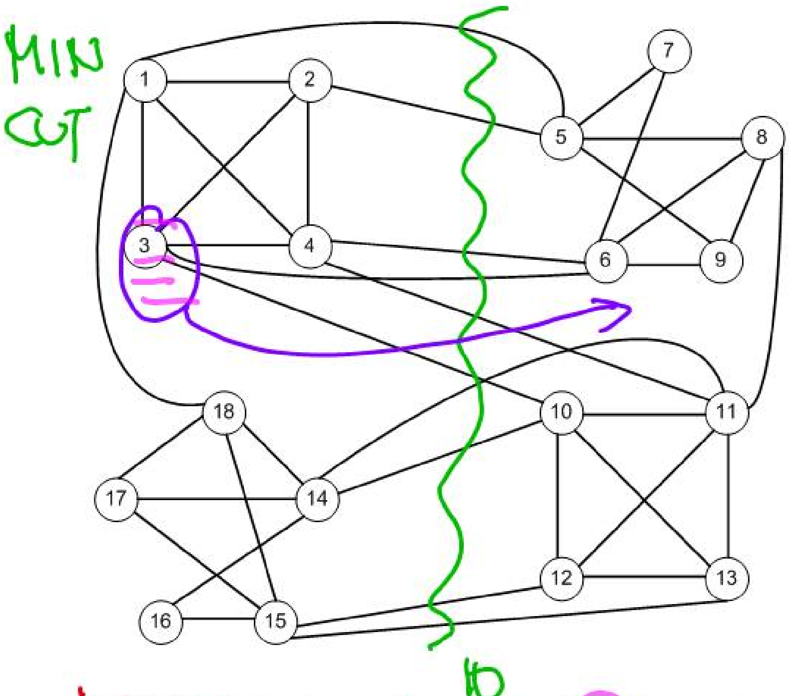
MIN 0

			Internal	3	3	2	4	3	3	4	3	3
			External	2	2	0	0	0	2	2	1	1
			Gain	1	1	2	4	3	1	2	2	2
Internal	External	Gain	**NODE**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
4	1	3	**1**	2	4	5	7	6	4	5	5	5
3	1	2	**2**	1	3	4	6	5	3	4	4	4
3	2	1	**3**	2	0	3	5	4	0	3	3	3
3	2	1	**4**	2	0	3	5	4	2	1	3	3
3	2	1	**14**	2	2	3	5	4	0	1	3	3
3	2	1	**15**	2	2	3	5	4	2	3	1	1
2	0	2	**16**	3	3	4	6	5	3	4	4	4
3	0	3	**17**	4	4	5	7	6	4	5	5	5
4	0	4	**18**	5	5	6	8	7	5	6	6	6



The min gain $= 0 \leqslant 2$: threshold

Switch nodes 3 & 10



Fiduccia - Mathyses (F&M)

"Move one node at a time"

Gain(i) = Internal - External

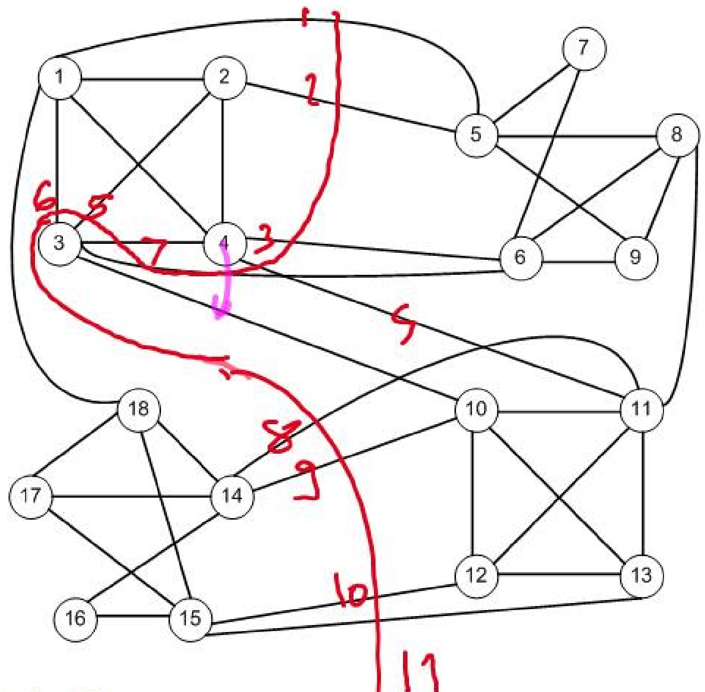
F&M Nongreedy with threshold 2

Tie Breaking Rule: "Smallest the first"

NODES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
GAIN	3	2	1	1	1	1	2	4	3	1	2	2	2	1	1	2	3	4

$\alpha = 1$ $\leq 2 \Rightarrow$ CONTINUE

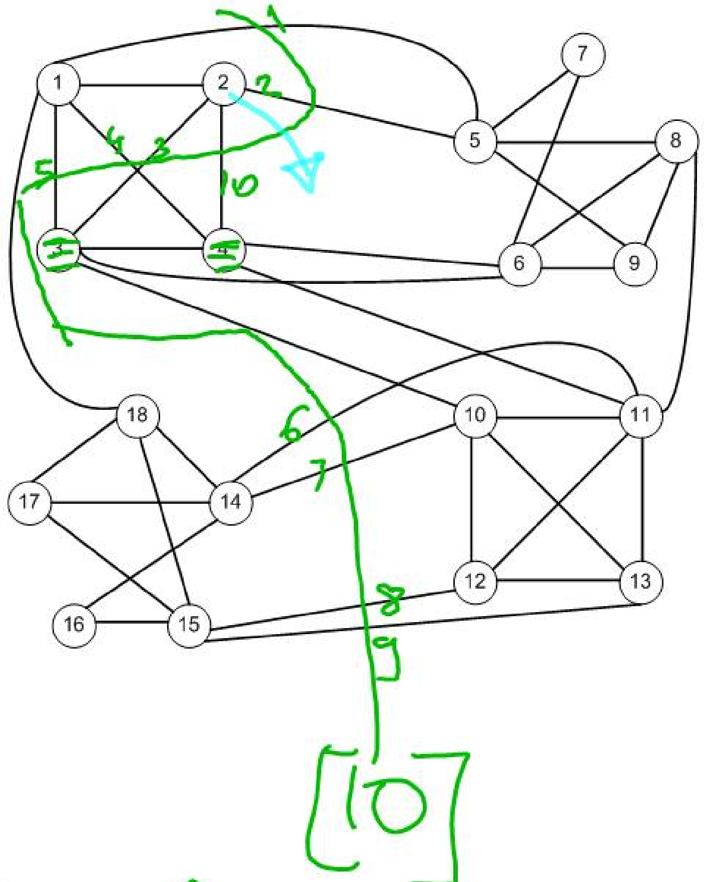
Freeze 3



Nodes	1	2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
GAIN	1	0	-1	1	3	2	4	3	3	2	2	2	1	1	2	3	4

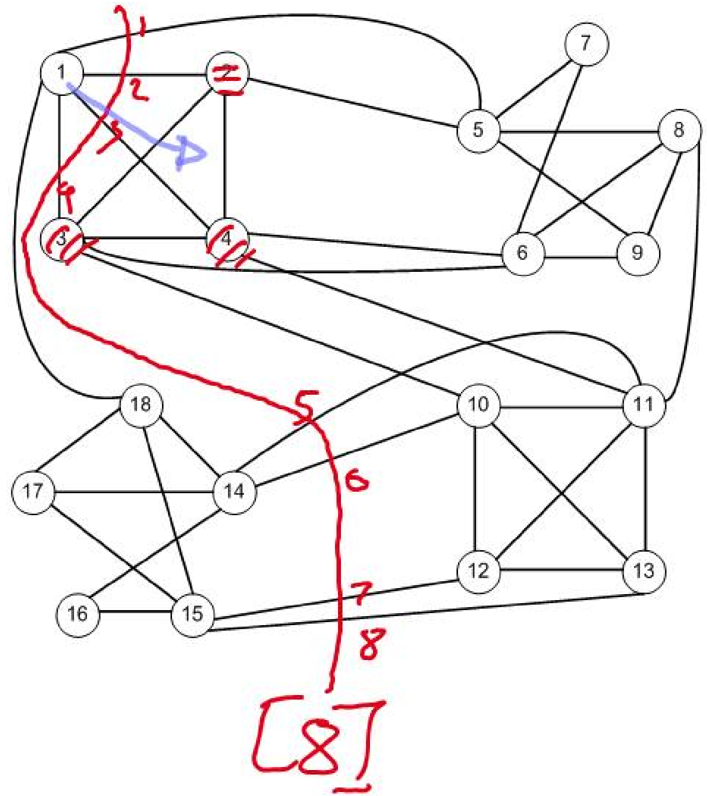
$\alpha = -1$

Freeze 4



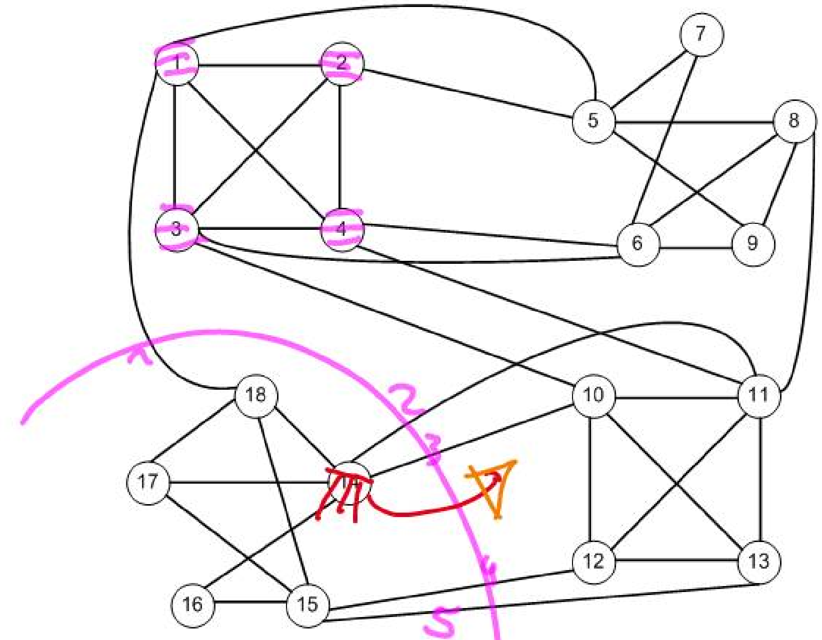
Nodes	1	2	5	6	7	8	9	10	11	12	13	14	15	16	17	18
GAIN	-1	-2	1	5	2	4	3	3	4	2	2	1	1	2	3	4

Freeze 2



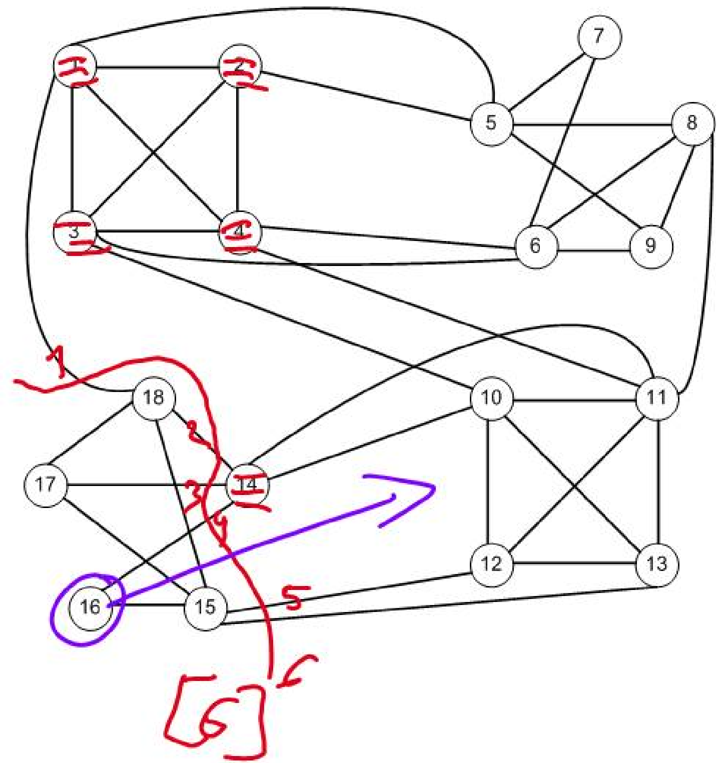
Nodes	1	5	6	7	8	9	10	11	12	13	14	15	16	17	18
GAIN	-3	3	5	2	4	3	3	4	2	2	1	1	2	3	4

$\alpha = -3$



Nodes	5	6	7	8	9	10	11	12	13	14	15	16	17	18	[5]
GAIN	5	5	2	4	3	3	4	2	2	1	1	2	3	2	

$\alpha = 1 \leq 2$: threshold



These iterations will ~~Continue~~ until

(i) all nodes are frozen

OR (ii) The min gain > threshold = 2

You must report to the best (not the final) value among all iterations

Tie

Nodes	5	6	7	8	9	10	11	12	13	15	16	17	18
GAIN	5	5	2	4	3	5	6	2	2	1	$\boxed{0}$	1	$\boxed{0}$

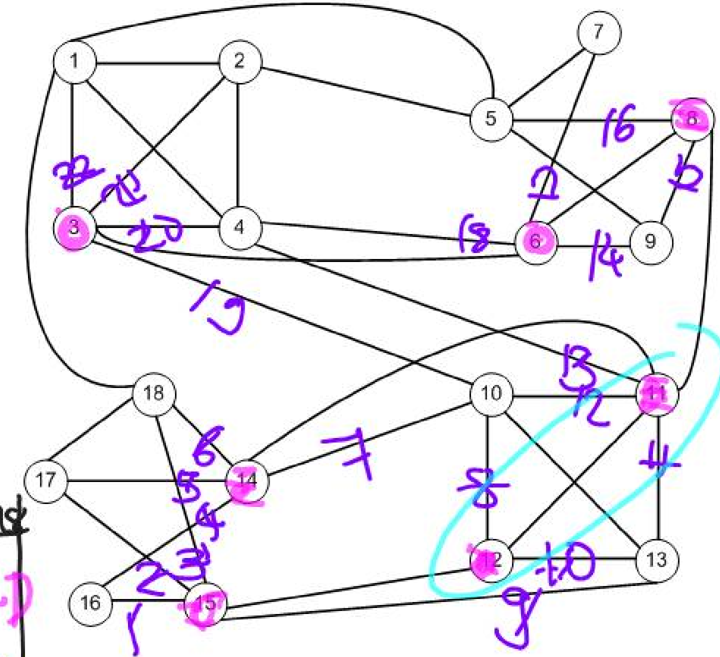
$\alpha = 0 \leq 2$ ✓

Apply F-M H. with thresold 1

$S = \{3, 6, 8, 11, 12, 14, 15\}$

for MINCUT

Break ties with lexicographical order



S	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
~~3~~, 6, 8, 11, 12, 14, 15	3	2	-3	-1	3	-1	0	0	-1	-3	0	0	-2	-3	-3	-2	-1	-1
~~6~~, 8, 11, 12, 14, 15	5	4		1	3	-3	0	0	-1	-1	0	0	-2	-3	-3	-2	-1	0
8, 11, 12, ~~14~~, 15	5	4		3	3		2	-2	1	-1	0	0	-2	-3	-3	-2	-1	0
8, 11, 12, ~~15~~	5	4		3	3		2	-2	1	1	-2	0	-2		-3	0	1	2
~~8~~, 11, 12	5	4		3	3		2	-2	1	1	-2	0				2	3	4
11, 12	5	4		5	5		2		3	1	-4	0				2	3	4

$22 \to 19 \to 16 \to 13$

$\to 9$

S	1	2	3	4	5	6	7	8	9
∅	5	10	10	7	15	15	12	8	6
5	5	2	10	5		5	8	2	6
3,5	-1	-2		-1		1	8	2	6
3,5,7	-1	-2		-1		-7		-6	2
3,5,7,9	-1	-2		-1		-13		-8	
1,3,5,7,9		-6		-1		-13		-8	
1,3,4,5,7,9		-10				-15		-8	

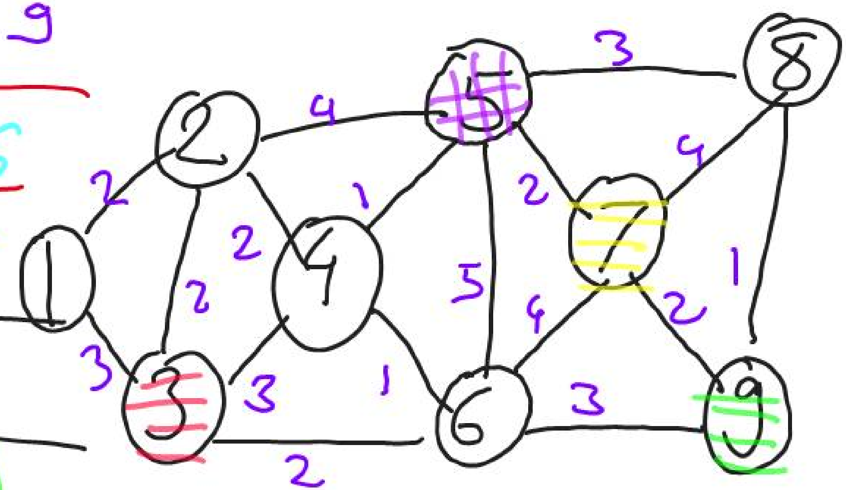
GREEDY STOPS

NONGREEDY STOPS

Nongreedy with threshold 2

Nongreedy's solution

S = {3, 5, 7, 9} with 35



Maxcut

S = ∅ cutsize 0

Edge weights

Tie Breaking: Smallest index