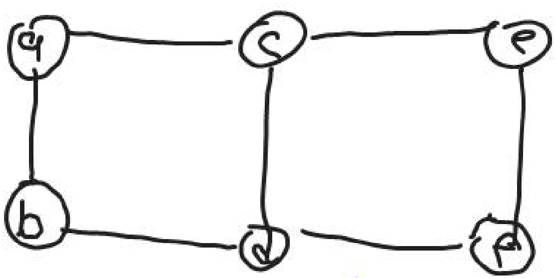
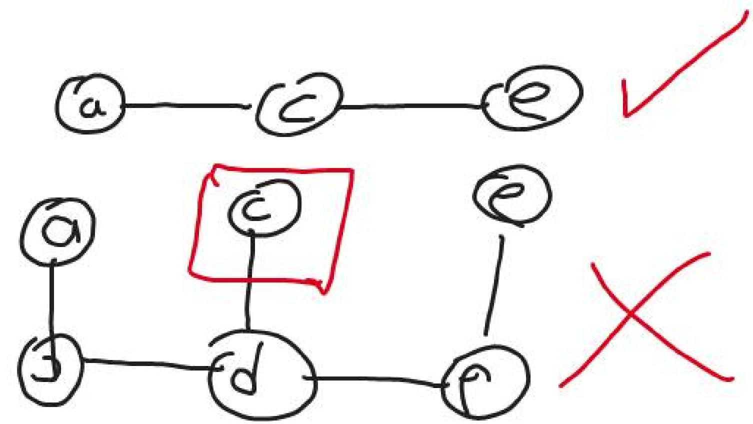
STEINER TREE

Given graph $G=(V,E)$ and $d: E \to \mathbb{R}_+$ distances and $R \subset V$, the shortest way of connecting all the vertices in R



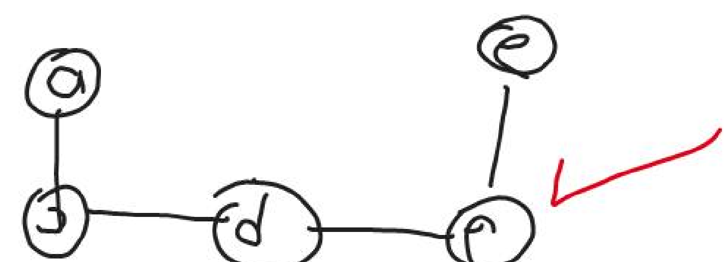
$R = \{a, e\}$



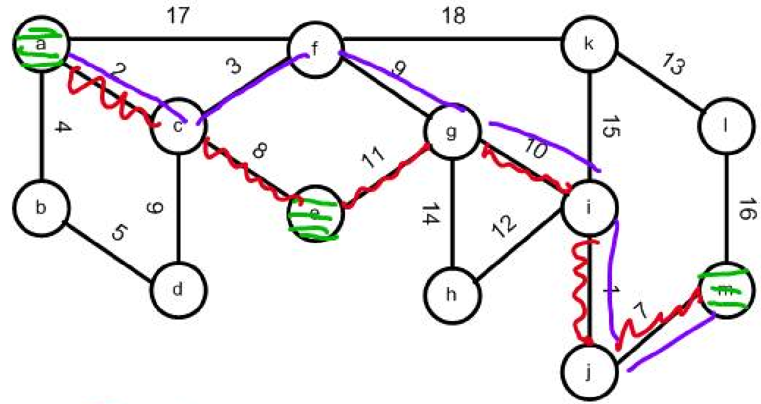
Special cases

(i) $R = V \leadsto$ spaning tree

(ii) $|R| = 2 \leadsto$ shortest path



$R = \{a, e, m\}$



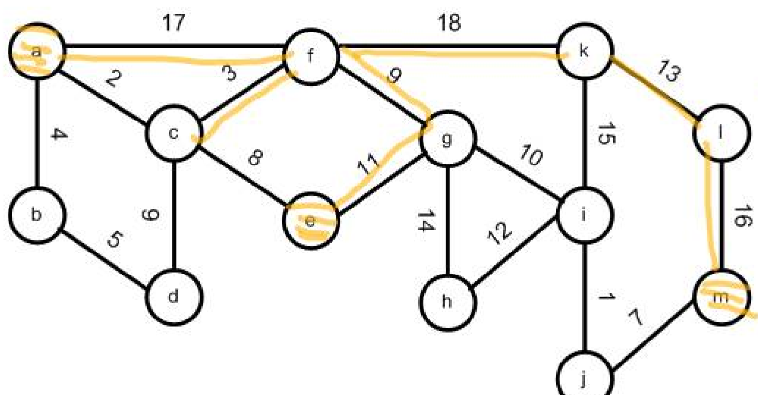
All leaves of S_tree are vertices in R (subset!)

S_tree : 39

~~S_tree~~ 32

Not feasible

e is not conected!

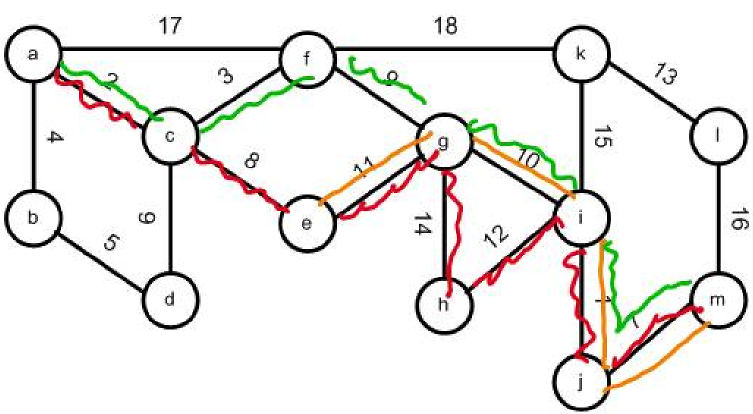


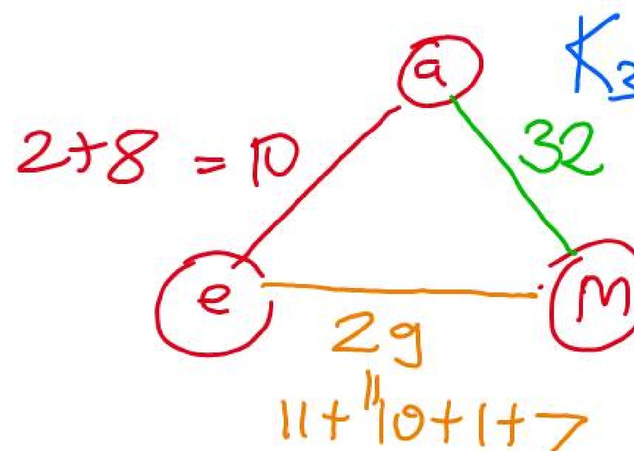
~~S_tree~~ : c is unnecessarily connected!

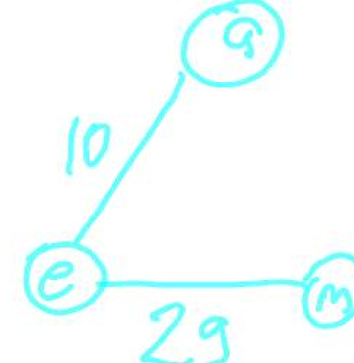
The opt. sol$^{\underline{n}}$ is hard to find like T.S.P. → Heuristics

A construction Heuristic for S-tree problem

S1. All pairwise shortest path problems among vertices in R
Construct a $\boxed{\text{complete graph}}$ whose lengths are shortest distances of the vertices in R $\longrightarrow$ $G1=(R, E_1)$

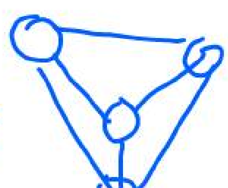


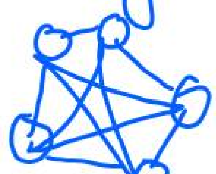




In a complete graph; for every pair of nodes, there is an edge.

K_4



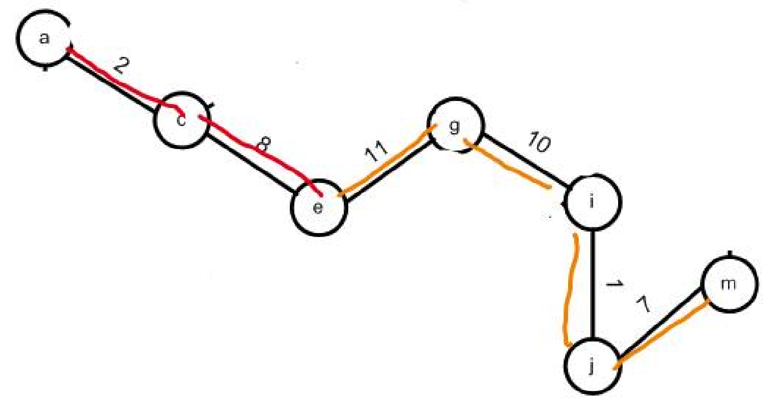


K_5

S2. Find MST in G_1 $\longrightarrow$ G_2
Min. Spanning Tree

S3. Expand the edges in G_2: using shortest paths in step 1

G2



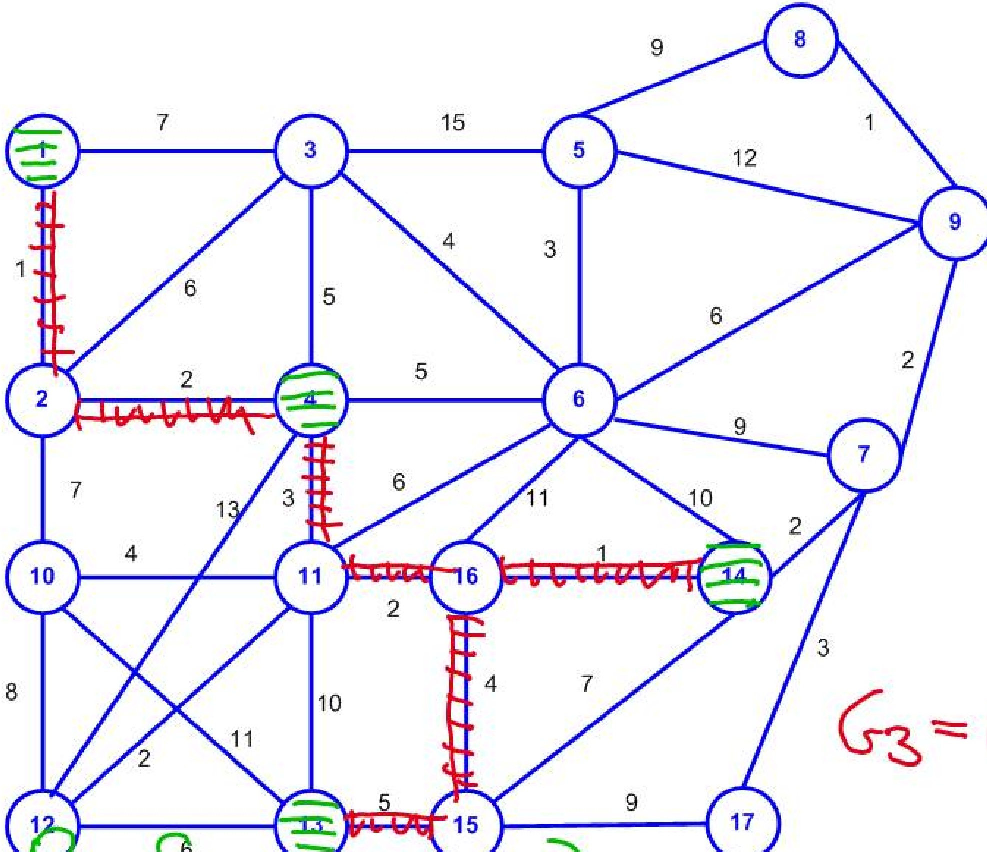
$\rightsquigarrow G_3$

$||$

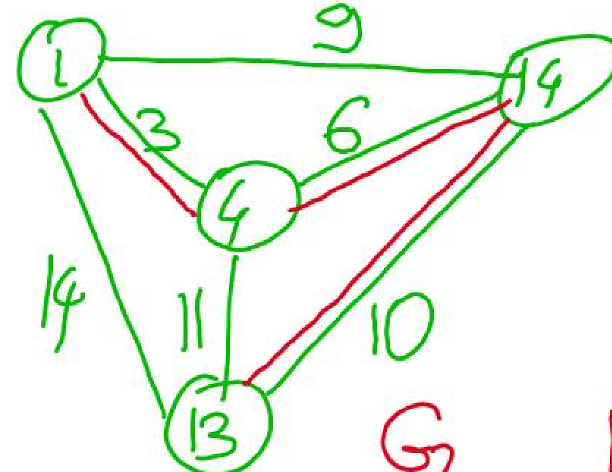
G_4

S4. Solve the MST in $G_3 \longrightarrow G_4$

$\rightsquigarrow$ length tree: 39



R = {1, 4, 13, 14}



G1

G2 19

G3 = G4: 18

(HYPER) GRAPH PARTITIONING

↳ a hyperedge may be incident to more than 2 nodes

→ BISECTION $V = V_1 \uplus V_2$ $\quad |V_1| = \left\lfloor \frac{|V|}{2} \right\rfloor$

$\gamma(V_1)$

$\gamma(V_2)$

$E = \delta(V_1;V_2) \uplus \gamma(V_1) \uplus \gamma(V_2)$

→ BIPARTITIONING $\quad V_1 \neq \emptyset \quad V_2 = V \setminus V_1$

$V_2 \neq \emptyset$

→ k-way $V = V_1 \uplus V_2 \uplus \cdots \uplus V_k$ $\quad V_i \neq \emptyset$

→ FREE PARTITION

k-way $\quad k$: variable

How many different k-way partitions are there, $|V| = n$?

$S(n,k)$

occupancy problem

Min edges in the cutset → MIN-CUT problem

Max → MAX-CUT

Min/max Cut problem: opt sol^n is hard to find like TSP, S-tree

$n = |V|$

2 → Construction Heuristics → Improvement

A) Bisection Problem
Bipartitioning

→ Half & Half

(i) $V_1 = \{1, \dots, \lfloor \frac{n}{2} \rfloor\}$

(ii) $V_1 = \{1, \dots, \lfloor \frac{n}{4} \rfloor\} \cup \{\lfloor \frac{3n}{4} \rfloor, \dots, n\}$

(iii) $V_1 = \{odd\}$
$V_2 = \{even \#\}$

→ Shortest Path

Fix two nodes s, t.

Shortest path from $s \to l_s(v)$
from $t \to l_t(v)$ $\forall v \in V \setminus \{s, t\}$

$V_1 = \{s\} \cup \{v \in V : l_s(v) < l_t(v)\}$

→ Randomized

Generate a random number (0,1) for $v \in V$

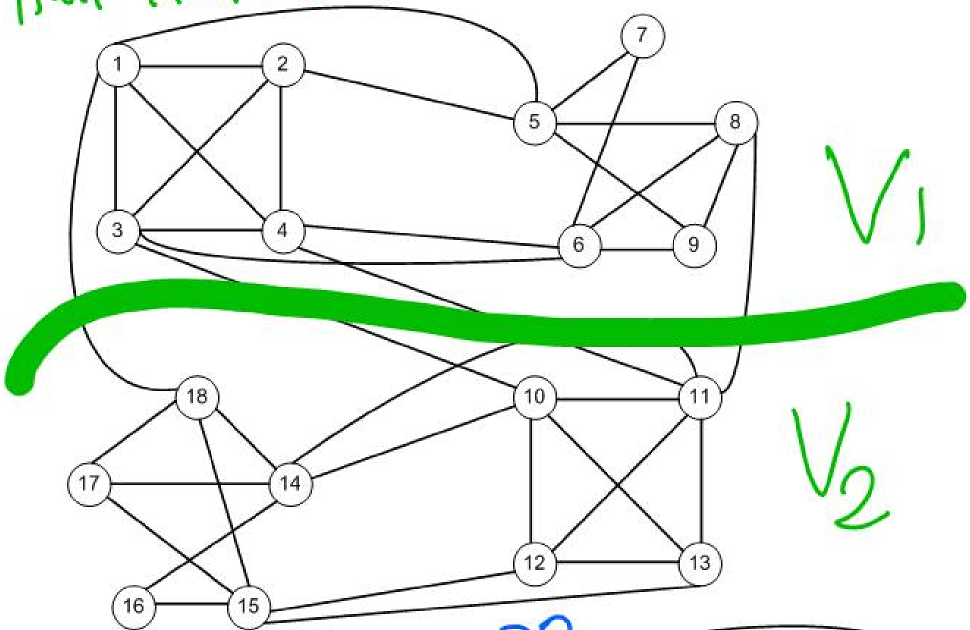
$r \in (0,1)$ if $r < 0.5$ $v \in V_1$
$v \in V_2$

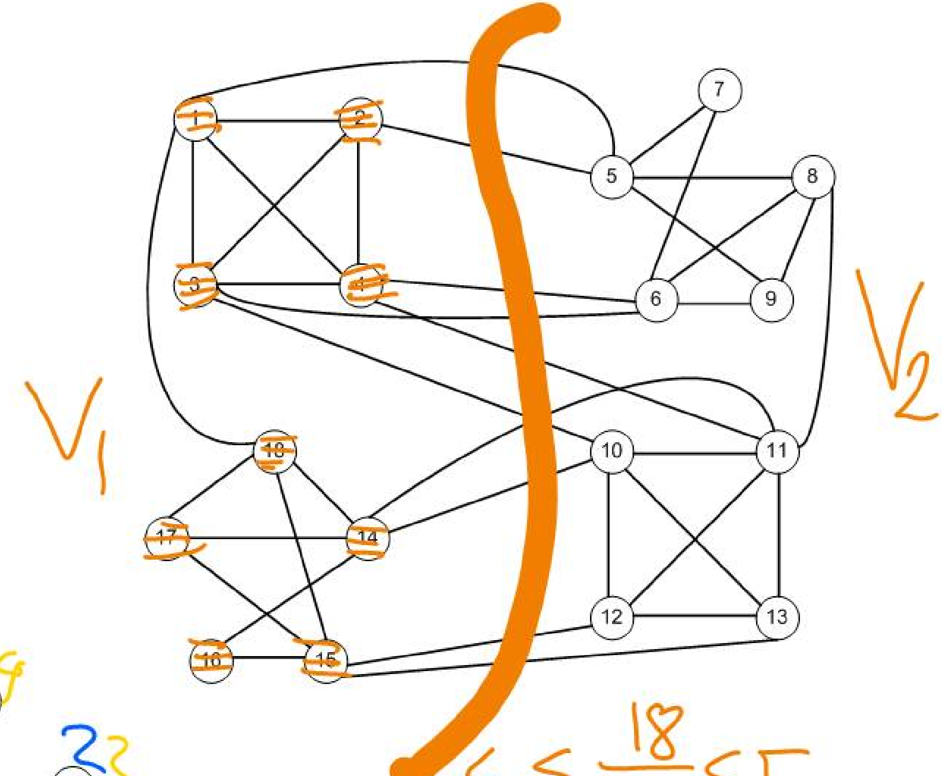
0.70 .43 .58 .22
.65 .34 .01 .79

$V = \{1, \dots, 8\}$

$V_1 = \{2, 4, 6, 7\}$
$V_2 = \{1, 3, 5, 8\}$

Half Half (i)

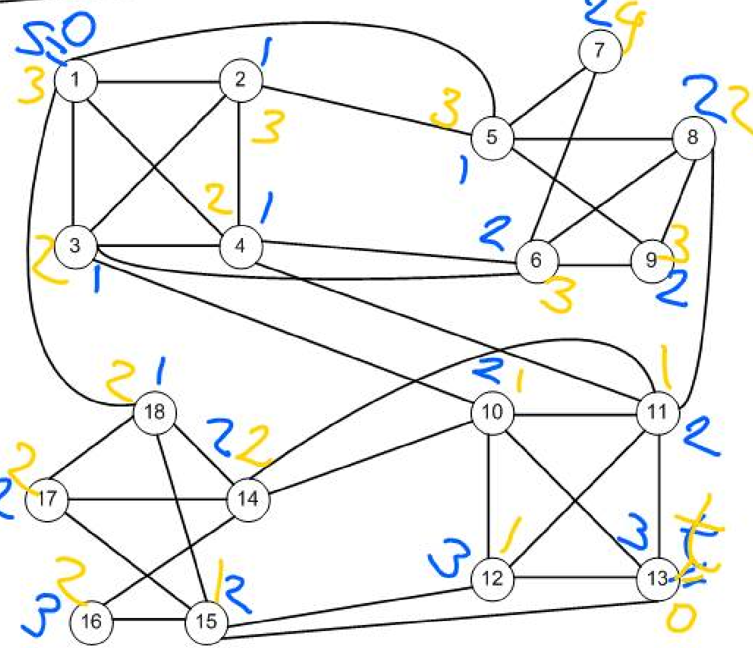




$$4 \le \frac{18}{4} \le 5$$

Shortest Path

$d_e = 1 \quad \forall e \in E$



$V_1 = \{1, 2, 3, 4, 5, 7, 6, 9, 18\}$

$V_2 = \{8, 10, 11, 12, 13, 14, 15, 16, 17\}$