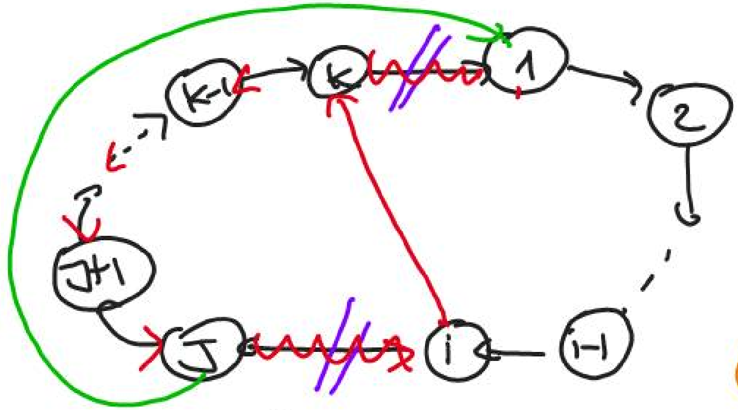
IMPROVEMENT H.

Given a tour → improve the tour

Choose a nonadjacent pair of edges

Suppose that we delete these edges

Get the new tour



GAIN: $(d_{k1} + d_{ij}) - (d_{ik} + d_{j1})$

— Initial tour

Calculate gains of <u>all</u> pairs of nonadjacent edges. If the max gain is negative STOP. Otherwise make the interchange & freeze

2-way interchange

2 → 3-way interchange

SPANNING TREE

Given $G=(V,E)$

$$x_{ij} = \begin{cases} 1 \text{ if } e=(i,j) \in T \\ 0, \text{ o/w} \end{cases}$$

Hard to formulate as an IP.

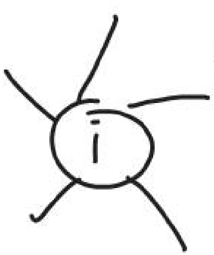
Easy to solve optimally

- Kruskal's
- Prim's

$$\text{Min} \sum_{e \in E} d_e x_e$$

s.t.

$$\sum_{e \in \delta(i)} x_e \geq 1 \quad \forall v \in V$$

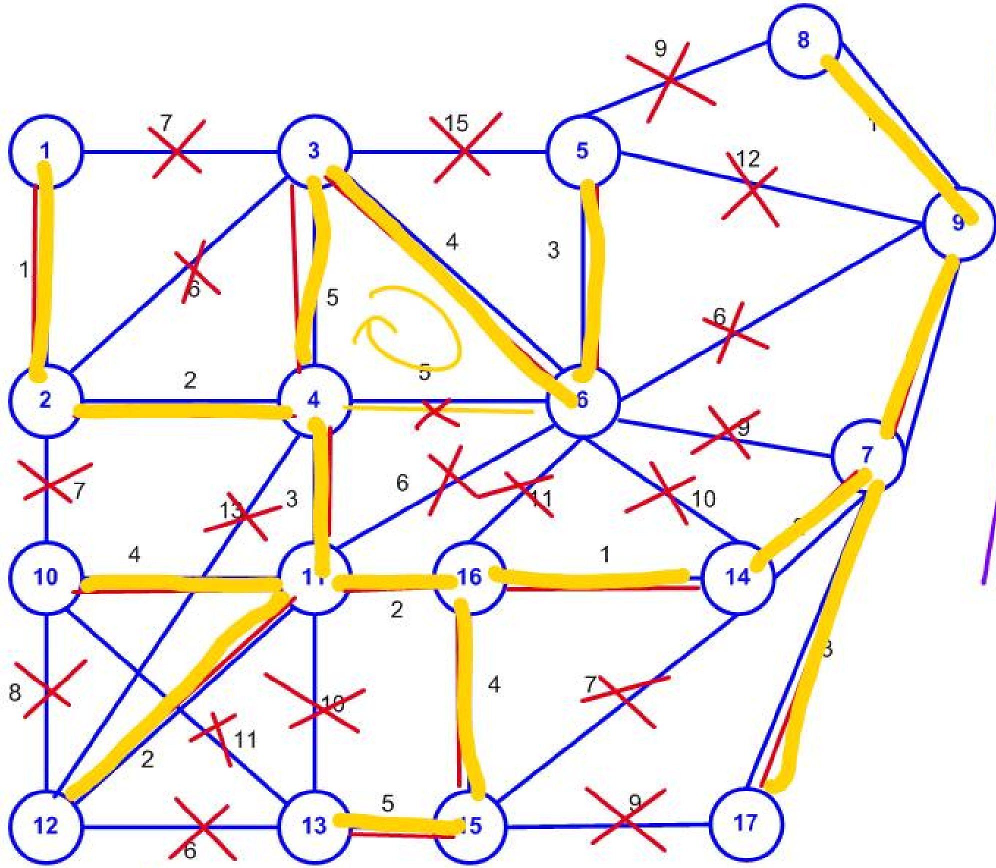


$$\sum_e x_e = n-1 \qquad n=|V|$$

Cycle prevention const

$$\sum_{e \in E(S)} x_e = |S|-1 \quad \forall S \subseteq V$$

$x_e = 0,1$



Length 44

LIST

1 2

8 9

14 16

⋮

3 5

Kruskal's

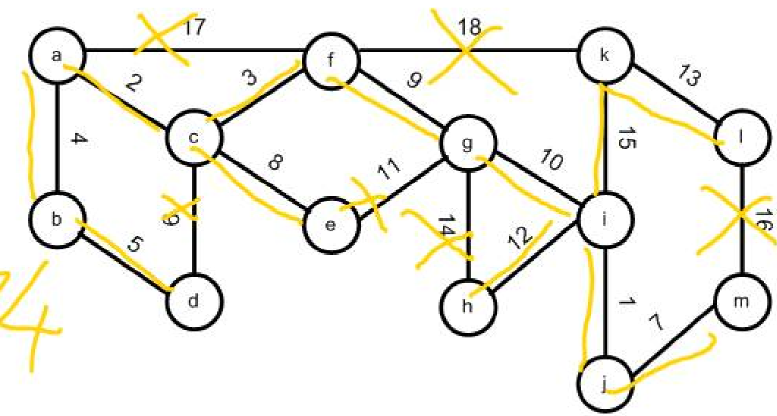
S1. Sort the edges in increasing order of lengths

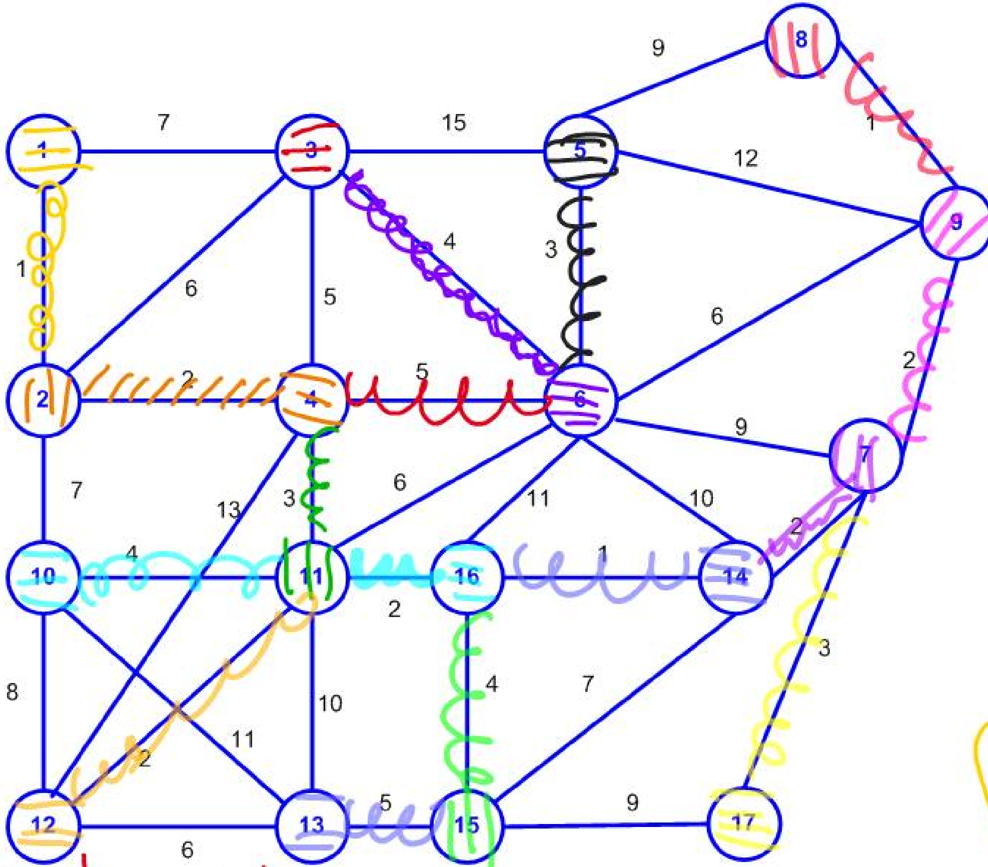
S2. wrt this order, examine edges

If an edge in the list creates a cycle ⟹ SKIP

o/w ADD

89? 74





$T^0 = \emptyset$

$T^1 = \{3\}$

$T^2 = \{3,6\}$

$T^3 = \{3,6,5\}$

$T^4 = \{3,6,5,4\}$

$T^5 = \{3,6,5,4\}$

$T^6 = \{3,5,6,5,4,2,1\}$

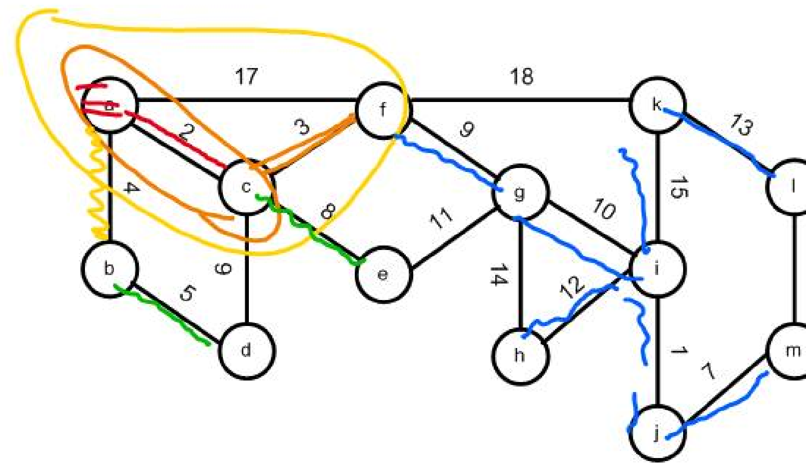
$T^7 = T^6 \cup \{11\}$

T^8 T^{12} T^{16}

T^9 T^{13}

T^{10} T^{14}

T^{11} T^{15}



PRIM's Algo

Start with any vertex. Connect the nearest unvisited vertex
⋮
until all vertices are connected