What is Combinatorics?

It is the art of counting without really counting.

It deals with
- Arrangements
- Selections
- Assignments
- Partitions

... etc

Combinatorics
- → Existence
  ∃ a particular configuration
- → Counting
  How many?
- → Optimization
  What is the best?

ASCII codes

Baku-Ceyhan-Tiflis Pipeline

New Campus NW Problem.

Examples will follow

A| assumption

∃ there exist

∋ such that

How to represent a language in a digital environment

digital → 0 or 1

A ... Z

a ... z

0 ... 9

. , ; : ' " ( ) { } [ ]

$ & @ ~ ... ^ > <

~ 200 symbols!

8 bits

1
2
4
8
16
32
64
128

"_ _ _ _ _ _ _ _" : BYTE

Min α ∋

$2^\alpha > 200$?

$2^8 : 256$ ←

ASCII ...
code

$37 = 1 \times 32 + 0 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$

100101

Counting

There are 40 segments in Baku-Tiflis-Ceyhan pipeline
In each segment, there are 7 different designs (ϕ)

How many different designs are there?

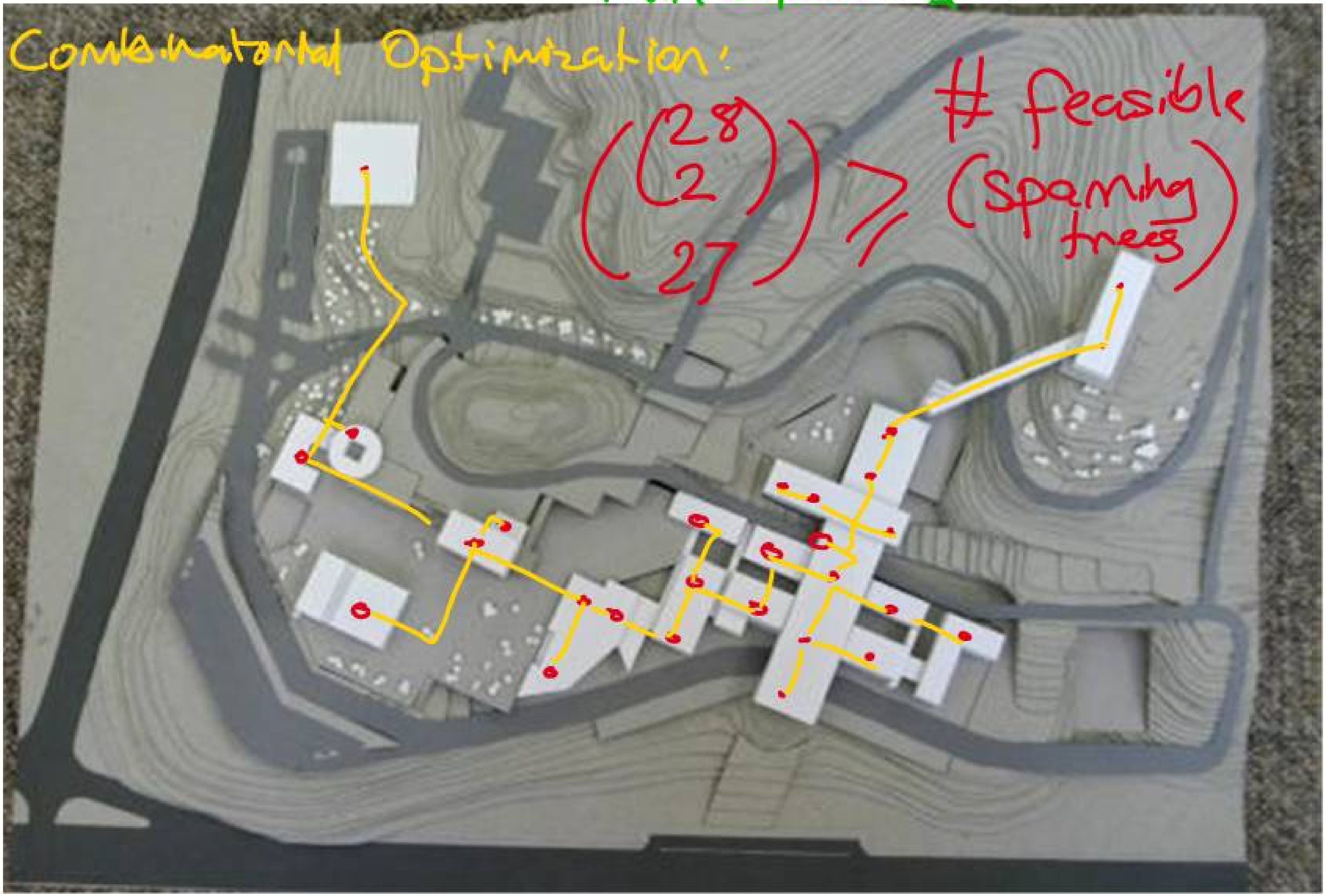
$$7(7)\cdots(7) = 7^{40} \cong 6 \times 10^{33}$$

A] 1) I can process 1 billion designs in a second
2) It takes 10^{17} years!

OPTIMIZATION

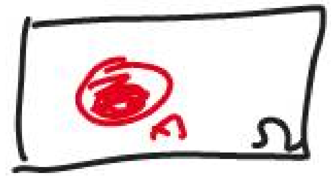
$\binom{28}{2}$: connections → 27 connections → to be selected

Min Spanning Tree 28 nodes



Counting — $n(A)$: # of occurrences of event (A) — Likelihood

COMBINATORICS — PROBABILITY



$$\frac{n(A)}{n(\Omega)} = P(A) = \text{prob of } A$$

$n(A) \longleftrightarrow P(A)$

Ω: sample space

How many solutions are there with the sum less than five if we roll a pair of dice

sum=2

(1 1) (1 2) (1 3) (1 4) (1 5) (1 6)

(2 1) (2 2) (2 3) (2 4) (2 5) (2 6)

(3 1) (3 2) (3 3) ...

(4 1) (4 2) (4 3) ...

(5 1) (5 2) (5 3) ...

(6 1) (6 2) (6 3) ...

$n(\Omega) = 6 \times 6 = 36$

$n(A) = 1 + 2 + 3 = 6$

$1+2+3+4+\cdots+98+99+100=?$

$+\ 100+99+98+\cdots+3+2+1$

$101+101+101+\cdots\ 101+101+101 = \dfrac{100(101)}{2}$

$1+2+\cdots+n = \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

$+\ n+(n-1)+\cdots+1$

$(n+1)+(n+1)+\cdots+(n+1) = \dfrac{n(n+1)}{2}$

Counting w/o Really counting one by one!

Rule of sum

If two events, A&B are ~~not~~ disjoint $n(A \text{ or } B) = n(A) + n(B)$ $- n(A \cap B)$

Rule of product

If two independent events A&B occur in seq then the # of different occur $= n(A) \times n(B)$

What is the probability of having (an ace or a red card) if I sample one card from the std. deck of 52?

A: an ace; B: a red card

$1/52$, $\frac{4+26}{52}$, $\frac{4+26-2}{52}$

$n(A) = 4$ $n(A \& B) = 2$

$n(B) = 26$

$n(A \text{ or } B) = 4 + 26 - 2 = 28$ $n(\Omega) = 52$

$P(A \text{ or } B) = \frac{28}{52}$

"without replacement"

What is the probability (of sampling 2 cards in sequence) of having $\underbrace{\text{an ace}}_{A}$ and a $\underbrace{\text{queen}}_{Q}$?

$\frac{16}{52}$, $\frac{1}{13} \cdot \frac{4}{51}$, $\frac{?\;\;16}{\binom{52}{2}}$

Rule of product

$n(\Omega) = \binom{52}{2}$

$n(A) = 4$

$n(Q) = 4$

with R.	W/O Rep.	
k-seq n^k	k-perm $\frac{n!}{(n-k)!} = P(n,k)$	(,) ARRANGE
(A, A) (B, A) (C, A) (A, B) (B, B) (C, B) (A, C) (B, C) (C, C)	(A, B) (B, A) (A, C) (C, A) (B, C) (C, B)	
k-multiset ? next week	k-comb $\binom{n}{k} = \frac{n!}{k!(n-k)!}$	SELECT { , }
{A, A} {A, B} {A, C} {B, B} {B, C} {C, C}	{A, B} {B, C} {C, A}	

SELECTION

ARRANGEMENT

E.G.

$n = 3$ {A, B, C}

$k = 2$

k-seq

If $n=k$

n	$n-1$		2	1
$n = k_1$	$n = k_2$	$\ldots$	$n = k_{k-1}$	$n = k_k$

stages

with — w/o

0000

k=n balls

$= n^k$

$n! = k!$

$n! = n(n-1)\cdots(3)\,2\,1$

$P(n;k)$

Permutation

$(n-k)$ missing cells

$n-k+1$

$$n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

$k<n$

n	$n-1$	$\ldots$	
n_1	n_2	$\ldots$	n_k

stages

R. of product

$$\underbrace{n(n)\cdots(n)}_{k} = n^k$$

Selection w/o rep.

The order of k cells is NOT important $= k!$

$$\binom{n}{k} = \frac{\frac{n!}{(n-k)!}}{k!} = \frac{n!}{k!(n-k)!}$$