Department of
Industrial Engineering

IE 454 Combinatorial Analysis

http://ie.cankaya.edu.tr/~kandiller/ie454/
Fall 2010 Tuesday 9:40-12:30 A201
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MIDTERM EXAM

1 page 1 side self hand-written cheat sheet is allowed!
14.12.2010 - 10:00

I hereby declare that I have neither given nor received any aid during the exam.
☐ *I accept* ☐ *I decline*
Signature:

Name: KEY & GRADING
Surname:
Number:

GRADE:

Question	Time	Points	Obtained
Q1	25	50	
Q2	25	40	
Q3	20	30	
Q4	25	45	
Q5	25	35	
TOTAL	120	200	

GOOD LUCK!

Q1. Generalized poker game:

A deck containing $4n$ ($n \geq 5$) cards has n cards each of four different suits, numbered, $1, 2, \ldots, n$ called the face value. In how many ways can five cards be chosen, so that they contain

[10 pt] five consecutive cards of the same suit?

Chose the suit $\binom{4}{1}$

The smallest card in the series of five conseqtive $(n-4)$ $\Big\}$ $4(n-4)$

[5 pt] four of the five cards with the same face value?

Choose face $\binom{n}{1}$

Choose all four $\binom{4}{4}$

Chose the remaining card $4(n-1)$ $\Big\}$ $4n(n-1)$

[5 pt] three cards with one face value and two cards with some other value?

Triplet: $\binom{n}{1}\binom{4}{3}$

Pair: $\binom{n-1}{1}\binom{4}{2}$ $\Big\}$ $= 12n(n-1)$

[5 pt] five cards of the same suit?

Choose the suit $\binom{4}{1}$

Choose 5 cards $\binom{n}{5}$ $\Big\}$ $4\binom{n}{5}$

[10 pt] five successively numbered cards?

Fix the smallest: $(n-4)$

Choose any face for 5 cards 4^5 $\Big\}$ $(n-4)4^5$

[5 pt] three of the five cards with the same face value?

Chose Triplet: $\binom{n}{1}\binom{4}{3}$

Chose the remaining two $\binom{4n-4}{2}$ $\Big\}$ $4n\binom{4n-4}{2}$

[10 pt] not more than two cards of the same suit?

$$12\binom{n}{2}^2 n \quad + \quad \binom{4}{1}\binom{n}{2}n^3$$

2 cards in two suits + one card in the remaining suit

2 cards in one suit
3 cards in different suits

Q2. Let a roll of three distinct dice produce a sum of n.

$x_1 + x_2 + x_3 = n$

$x_i = 1, \dots, 6$

[5 pt] Find bounds (lower, upper) on n.

$3 \leq n \leq 18$

[10 pt] Construct the generating function.

$$(x + x^2 + \cdots + x^6)^3 = x^3 \, (1 + x + \cdots + x^5)^3 =$$

$$= x^3 \underbrace{[1 - x^6]^3}_{k(x)} \underbrace{[1 + x + x^2 + \cdots]^3}_{\ell(x)}$$

BINOMIAL MULTISET

[25 pt] Find the probabilities for n=5 and for n=11.

2^{nd} and 8^{th} convolution of $k(x) \cdot \ell(x)$

$$k(x) = \binom{3}{0} - \binom{3}{1}x^6 + \binom{3}{2}x^{12} - \binom{3}{3}x^{18} \quad [5]$$

$$\ell(x) = \binom{2+0}{0} + \binom{3}{1}x + \cdots + \binom{2+k}{k}x^k + \cdots \quad [5]$$

2^{nd} convolution : $\binom{3}{0}\binom{4}{2} = 6$ [5]

8^{th} convolution : $\binom{3}{0}\binom{10}{8} - \binom{3}{1}\binom{4}{2} = 45 - 18 = 27$ [5]

Probabilities: $\dfrac{6}{216}$, $\dfrac{27}{216}$ [5pt]

and classify

Q3. Find a recurrence relation for the number of regions into which the plane is divided by n straight lines if every pair of lines intersect, but no three lines meet at a common point. Solve for a_n. What is a_{100}?

$a_1=2$, $a_2=4$, $a_3=7$, $a_4=11$, ..., $a_n = a_{n-1}+n$ [9] $n=2,3,\dots$

$a_1=2 \Rightarrow$ Boundary condition (1)

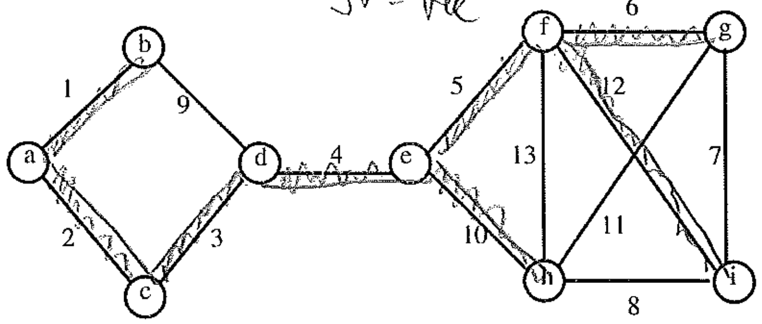
Non homogeneous, constant coefficient RR with depth 1 [5]

$a_n = n + a_{n-1} = n+(n-1)+a_{n-2} = \cdots = n+(n-1)+\cdots+3+2+a_1$

$a_n = [1+2+\cdots+n]+1 = \frac{n(n-1)}{2}+1$ [10]

$a_{100} = 50(99)+1 = 4951$ [5]

SP-Tree



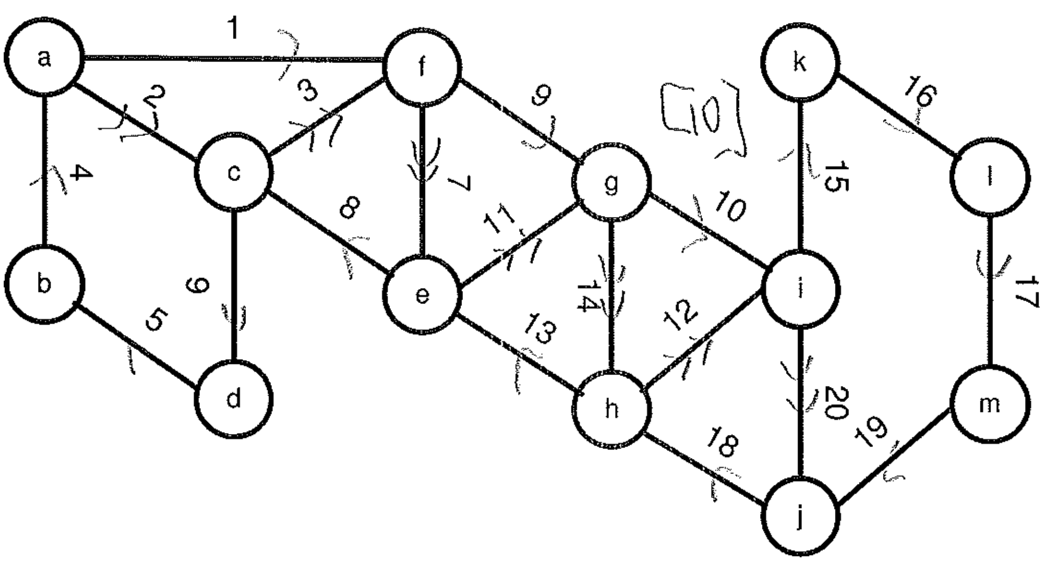
Q4.

Solve the shortest path problems from node a using the modified BFS method.

Iter No	Node	Parent	Distance from(a)	Neighbor Nodes	Parent Update	Label Update	Data Structure
1	a	none	0	b, c	p(b)=a p(c)=a	l(b)=1 l(c)=2	c b ~~a~~
2	b	a	1	a, d	p(d)=b	l(d)=10	d c ~~b~~
3	c	a	2	a, d	p(d)=c	l(d)=5	d c
4	d	c	5	c, b, e	p(e)=d	l(e)=9	e ~~d~~
5	e	d	9	d, f, h	p(f)=e p(h)=e	l(f)=14 l(h)=19	h f ~~e~~
6	f	e	14	e, g, i, h	p(g)=f p(i)=f	l(g)=20 l(i)=26	i g h ~~f~~
7	h	e	19	i, e, g, f	—	—	i g ~~h~~
8	g	f	20	f, i, h	—	—	i ~~g~~

Each [5pt]

Shortest path tree[5 pt]:



Q5. (a) Does there exist a Eulerian circuit? Why? If so, find the Eulerian tour.

[10] deg(a) = deg(j) = 3 odd ∴ ∄ no Eulerian tour!

(b) Does there exist a Eulerian path? Why? If so, find the Eulerian path. (on the graph)

[5] Yes, there are only two nodes with odd degree (a) & (j)

Cycle 1 $(a) \xrightarrow{1} (f) \xrightarrow{9} (g) \xrightarrow{10} (i) \xrightarrow{15} (k) \xrightarrow{16} (l) \xrightarrow{17} (m)$

$\xrightarrow{4} (a)$, $(b) \xleftarrow{5} (d) \xleftarrow{9} (c) \xleftarrow{8} (e) \xleftarrow{13} (h) \xleftarrow{18} (j) \xleftarrow{19}$ [10]

Path $(a) \xrightarrow{2} (c) \xrightarrow{3} (f) \xrightarrow{7} (e) \xrightarrow{11} (g) \xrightarrow{14} (h) \xrightarrow{12} (i) \xrightarrow{20} (j)$