Department of
Industrial Engineering

IE 454 Combinatorial Analysis
http://ie454.cankaya.edu.tr
FALL 2010 Tuesday 9:40-12:30 A201
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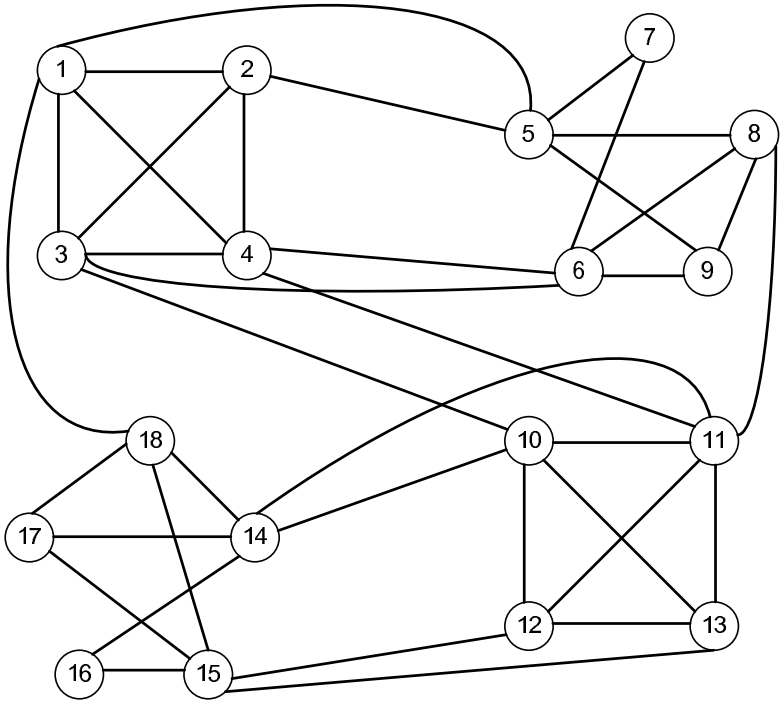
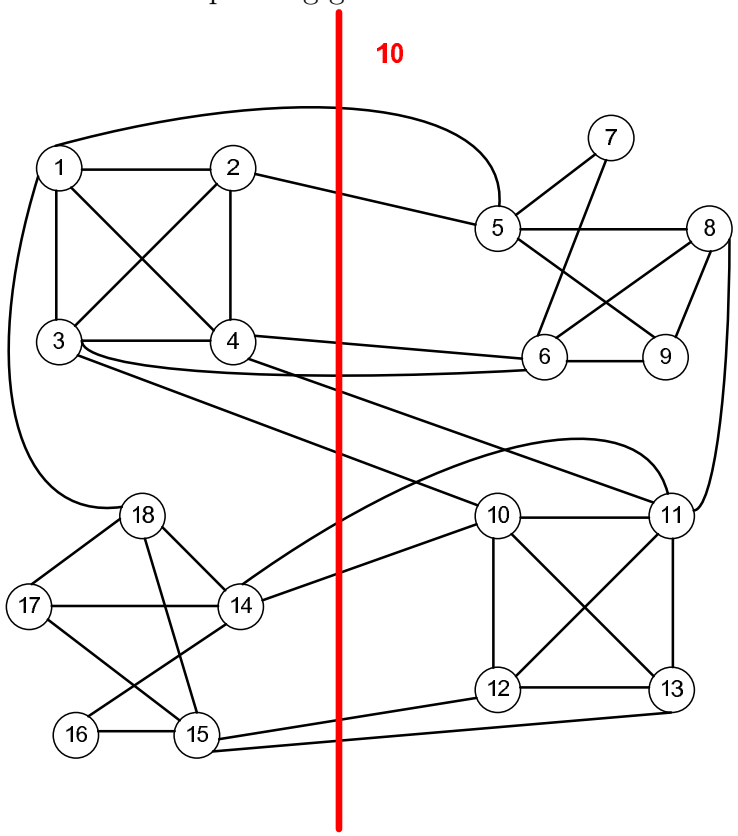
Solution to HOMEWORK 4

Given the below graph with equal edge weights, determine the Kernighan–Lin solution of the maxcut/mincut problem if the initial partition is $S = \{1, 2, 3, 4, 14, 15, 16, 17, 18\}$.



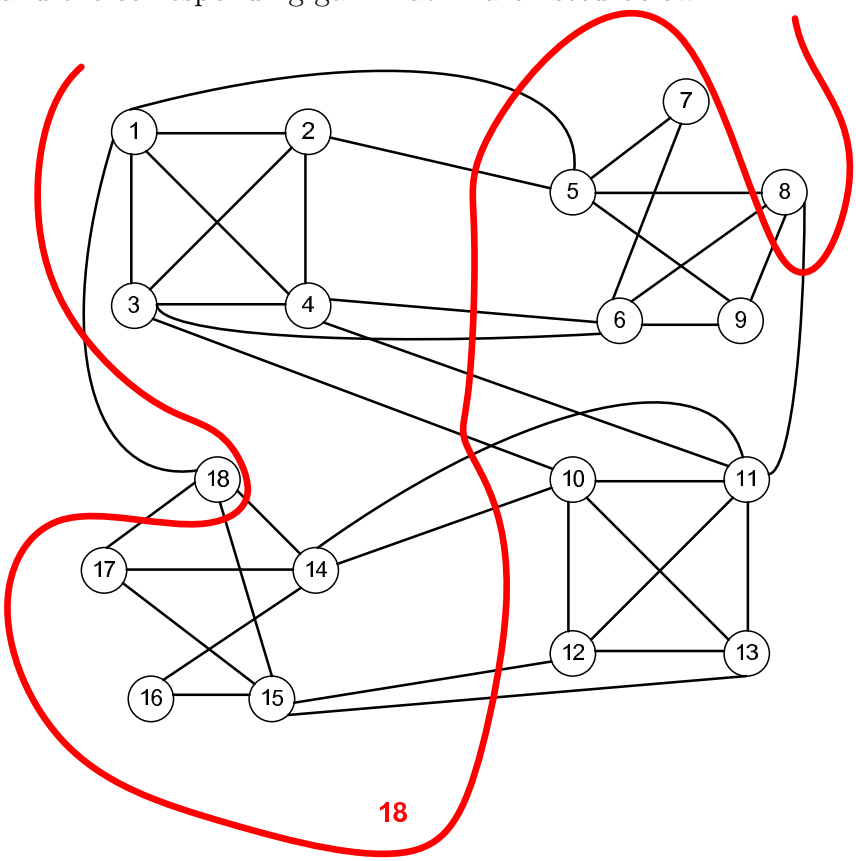
maxcut The initial solution and the corresponding gain matrix are listed below:



ITERATION 1												
MAX	8		Internal	3	3	2	4	3	3	4	3	3
			External	2	2	0	0	0	2	2	1	1
			Gain	1	1	2	4	3	1	2	2	2
Internal	External	Gain	**NODE**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
4	1	3	**1**	2	4	5	7	6	4	5	5	5
3	1	2	**2**	1	3	4	6	5	3	4	4	4
3	2	1	**3**	2	0	3	5	4	0	3	3	3
3	2	1	**4**	2	0	3	5	4	2	1	3	3
3	2	1	**14**	2	2	3	5	4	0	1	3	3
3	2	1	**15**	2	2	3	5	4	2	3	1	1
2	0	2	**16**	3	3	4	6	5	3	4	4	4
3	0	3	**17**	4	4	5	7	6	4	5	5	5
4	0	4	**18**	5	5	6	8	7	5	6	6	6

The best gain is 8 by means of interchanging nodes 8 and 18. Freeze them, the next solution has value 10+8.

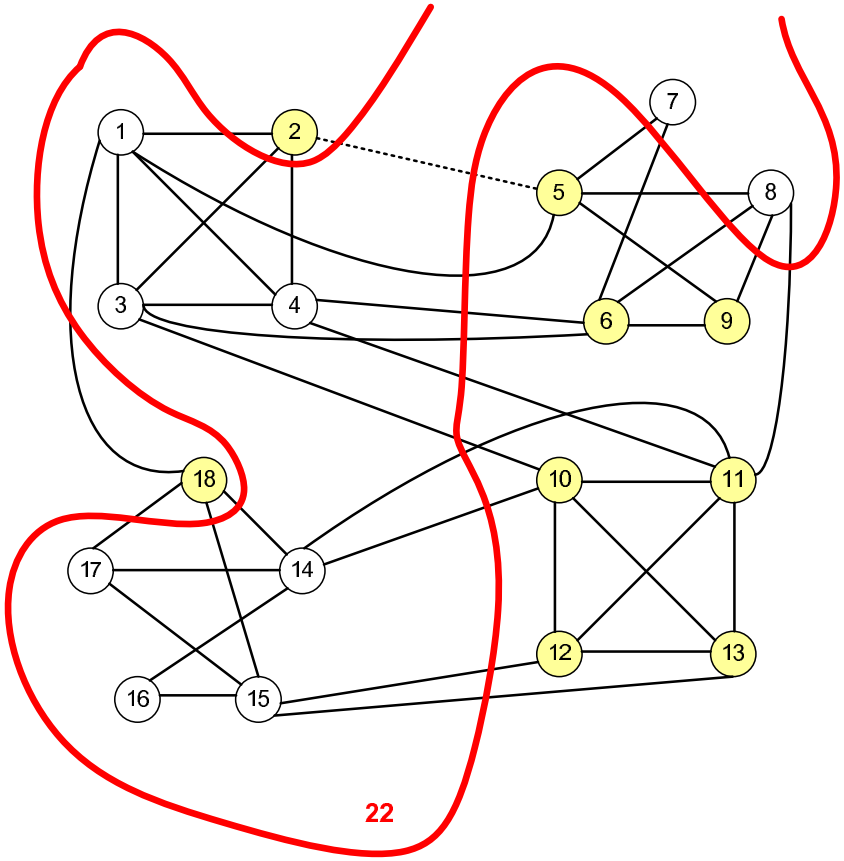
The next solution and the corresponding gain matrix are listed below:



ITERATION 2			Internal	2	2	2	4	2	3	3	3	3
MAX	4		External	3	3	0	0	1	2	3	1	1
			Gain	-1	-1	2	4	1	1	0	2	2
Internal	External	Gain	NODE	5	6	7	8	9	10	11	12	13
3	2	1	1	-2	0	3	5	2	2	1	3	3
3	1	2	2	-1	1	4	6	3	3	2	4	4
3	2	1	3	0	-2	3	5	2	0	1	3	3
3	2	1	4	0	-2	3	5	2	2	-1	3	3
2	3	-1	14	-2	-2	1	3	0	-2	-3	1	1
2	3	-1	15	-2	-2	1	3	0	0	-1	-1	-1
2	0	2	16	1	1	4	6	3	3	2	4	4
2	1	1	17	0	0	3	5	2	2	1	3	3
4	0	4	18	5	5	6	8	7	5	6	6	6

The best gain is 4 by means of interchanging nodes 7 and 2. Freeze them, the next solution has value 18+4.

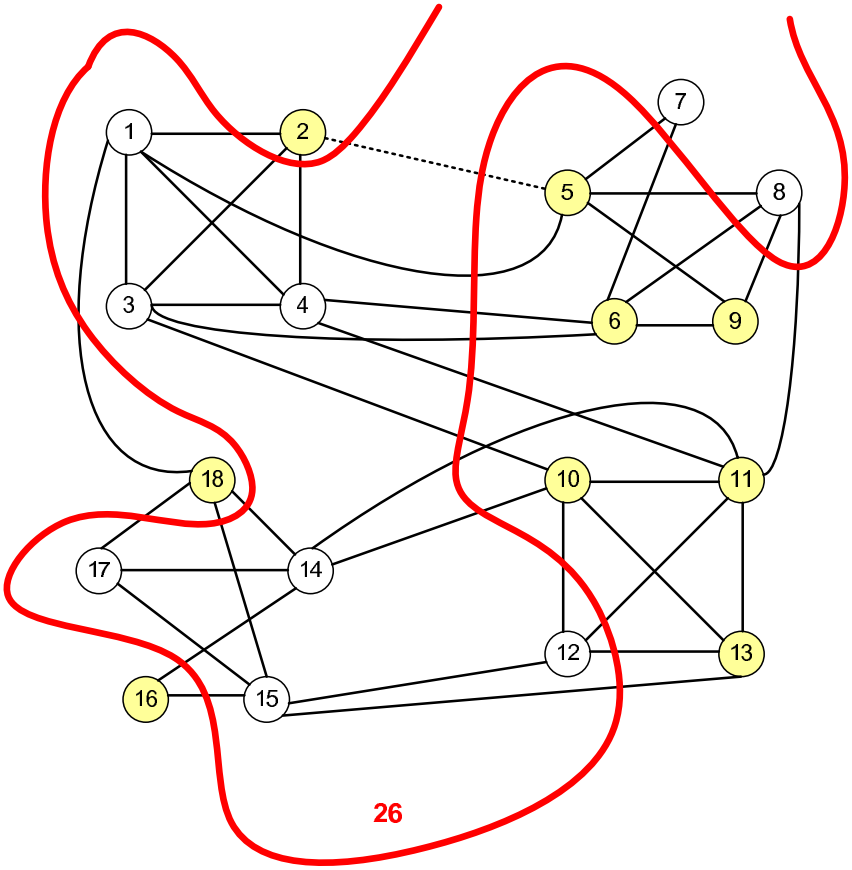
The next solution and the corresponding gain matrix are listed below:



ITERATION 3			Internal	2	1	2	4	2	3	3	3	3
MAX	4		External	3	4	0	0	1	2	3	1	1
			Gain	-1	-3	2	4	1	1	0	2	2
Internal	External	Gain	NODE	5	6	7	8	9	10	11	12	13
2	3	-1	1	-4	-4	1	3	0	0	-1	1	1
3	1	2	2	-1	-1	4	6	3	3	2	4	4
2	3	-1	3	-2	-6	1	3	0	-2	-1	1	1
2	3	-1	4	-2	-6	1	3	0	0	-3	1	1
2	3	-1	14	-2	-4	1	3	0	-2	-3	1	1
2	3	-1	15	-2	-4	1	3	0	0	-1	-1	-1
2	0	2	16	1	-1	4	6	3	3	2	4	4
2	1	1	17	0	-2	3	5	2	2	1	3	3
4	0	4	18	5	5	6	8	7	5	6	6	6

The best gain is 8 by means of interchanging nodes 12 and 16. Freeze them, the next solution has value 22+4.

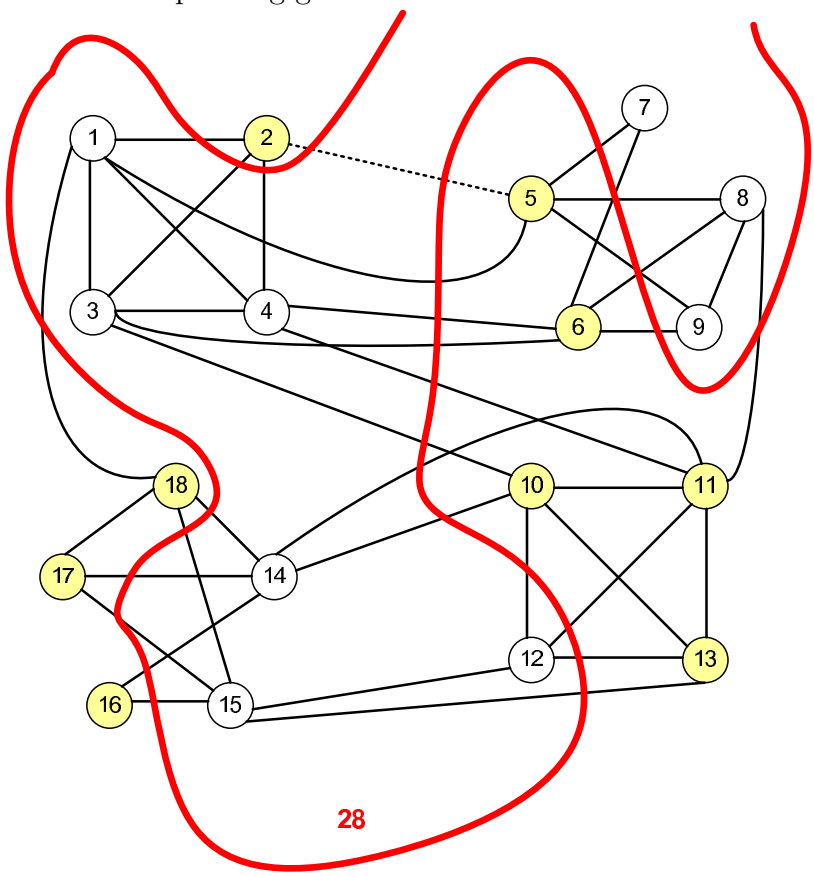
The next solution and the corresponding gain matrix are listed below:



ITERATION 4			Internal	2	1	2	4	2	2	2	3	2
MAX	2		External	3	4	0	0	1	3	4	1	2
			Gain	-1	-3	2	4	1	-1	-2	2	0
Internal	External	Gain	**NODE**	**5**	**6**	**7**	8	**9**	**10**	**11**	**12**	**13**
2	3	-1	**1**	-4	-4	1	3	0	-2	-3	1	-1
3	1	2	**2**	-1	-1	4	6	3	1	0	4	2
2	3	-1	**3**	-2	-6	1	3	0	-4	-3	1	-1
2	3	-1	**4**	-2	-6	1	3	0	-2	-5	1	-1
1	4	-3	**14**	-4	-6	-1	1	-2	-6	-7	-1	-3
2	3	-1	**15**	-2	-4	1	3	0	-2	-3	-1	-3
2	0	2	**16**	1	-1	4	6	3	1	0	4	2
2	1	1	**17**	0	-2	3	5	2	0	-1	3	1
4	0	4	**18**	3	1	6	8	5	3	2	6	4

The best gain is 2 by means of interchanging nodes 9 and 17. Freeze them, the next solution has value 26+2.

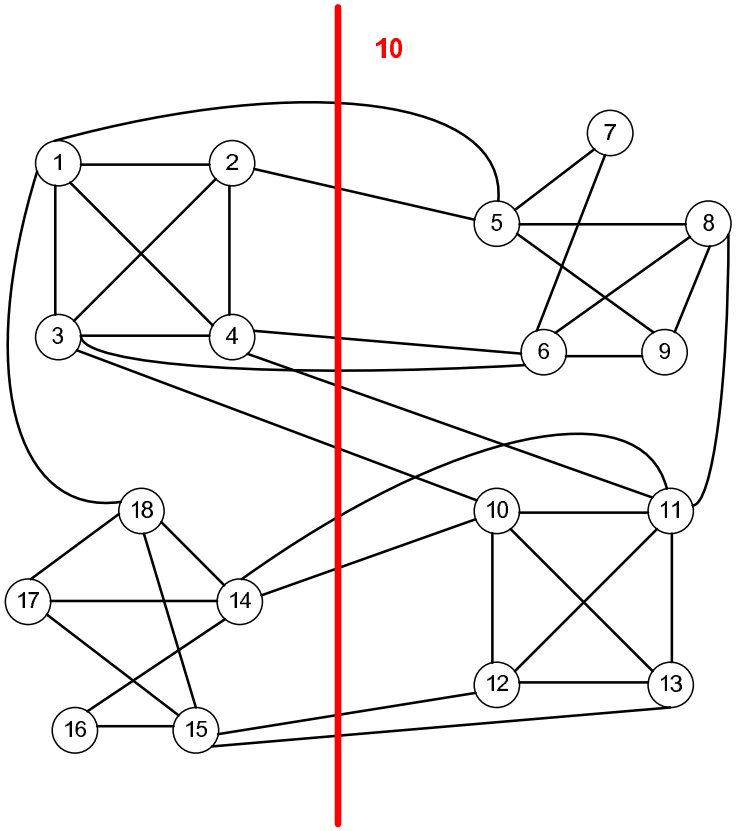
The next solution and the corresponding gain matrix are listed below:



ITERATION 5			Internal	1	0	2	4	2	2	2	3	2
MAX	-1		External	4	5	0	0	1	3	4	1	2
			Gain	-3	-5	2	4	1	-1	-2	2	0
Internal	External	Gain	**NODE**	**5**	**6**	**7**	8	9	**10**	**11**	**12**	**13**
2	3	-1	**1**	-6	-6	1	3	0	-2	-3	1	-1
3	1	2	**2**	-3	-3	4	6	3	1	0	4	2
2	3	-1	**3**	-4	-8	1	3	0	-4	-3	1	-1
2	3	-1	**4**	-4	-8	1	3	0	-2	-5	1	-1
0	5	-5	**14**	-8	-10	-3	-1	-4	-8	-9	-3	-5
1	4	-3	**15**	-6	-8	-1	1	-2	-4	-5	-3	-5
2	0	2	**16**	-1	-3	4	6	3	1	0	4	2
2	1	1	**17**	-2	-4	3	5	2	0	-1	3	1
4	0	4	**18**	1	-1	6	8	5	3	2	6	4

The maximum gain is negative. STOP!

mincut The initial solution and the corresponding gain matrix are listed below:



ITERATION 1												
MAX	8		Internal	3	3	2	4	3	3	4	3	3
			External	2	2	0	0	0	2	2	1	1
			Gain	1	1	2	4	3	1	2	2	2
Internal	External	Gain	**NODE**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
4	1	3	**1**	2	4	5	7	6	4	5	5	5
3	1	2	**2**	1	3	4	6	5	3	4	4	4
3	2	1	**3**	2	0	3	5	4	0	3	3	3
3	2	1	**4**	2	0	3	5	4	2	1	3	3
3	2	1	**14**	2	2	3	5	4	0	1	3	3
3	2	1	**15**	2	2	3	5	4	2	3	1	1
2	0	2	**16**	3	3	4	6	5	3	4	4	4
3	0	3	**17**	4	4	5	7	6	4	5	5	5
4	0	4	**18**	5	5	6	8	7	5	6	6	6

The minimum gain is 0. Thus STOP if you are not after alternative optimum solutions.