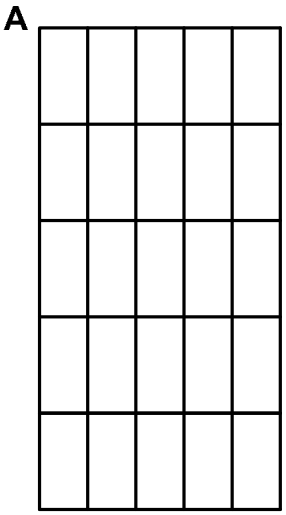
Department of
Industrial Engineering

IE 454 Combinatorial Analysis

http://ie454.cankaya.edu.tr
Fall 2010 Tuesday 9:40-12:30 A201
Levent Kandiller
kandiller@cankaya.edu.tr
Voice: 189 Dean's office

HOMEWORK 2 (Due: Nov. 23)

Q1.



(a) How many different ways are there to pick 4 different cells in this 5 by 5 mesh?

(b) How many different ways are there to pick 4 cells in sequence as 1^{st},2^{nd},3^{rd}, and 4^{th} such that one can pick the same cell in at most four times in this 5 by 5 mesh?

(c) How many different ways are there to pick 15 cells in this 5 by 5 mesh such that exactly 3 cells in each column are selected?

(d) How many different ways are there to pick 3 cells in this 5 by 5 mesh such that no two cells in the same row and column are selected?

(e) Consider that you are at point B. The side figure is the road-map of a district. You may use horizontal or vertical streets. How many different routes of length 9 from B to A are there?

Q2. *Capacitated bins and indistinguishable balls:*
Consider n indistinguishable balls and m bins, where each bin has a capacity of $c(i)$ $(i = 1,\ldots,m)$ balls. Note that $n \leq \sum_i c(i)$; one or more bins may have room for all n balls; we don't care which balls are in which bins, nor do we distinguish between positions in the bins; and bins need not be occupied.
Let $N(k)$ denotes the number of ways of packing k balls into m bins with capacities $c(i)$.

(a) If we have 5 bins with capacities 3,2,5,4,2 respectively, then what will be $N(1)$?

(b) If we have 5 bins with capacities 3,2,5,4,2 respectively, then what will be $N(2)$?

(c) If we have 5 bins with capacities 3,2,5,4,2 respectively, then what will be $N(14)$?

(d) Prove $N(k) = N(c-k)$ combinatorially, where c is the total capacity $c(1)+c(2)+\cdots+c(m)$:

(e) If we have 5 bins with capacities 3,2,5,4,2 respectively, then what will be $\sum_{k=0}^{16} N(k)$?

(f) What is $\sum_{k=0}^{c} N(k)$ in general, where c is the total capacity?

(g) In how many ways can the n balls be distributed in the m bins? [BONUS]

Q3.

$$x_1 + x_2 + x_3 + x_4 + x_5 = n$$

$$x_1, x_2, x_3, x_4 = 1, 2, 3, 4, 5, 6;\ 0 \leq x_5 < 6 \text{(Strict inequality!)}.$$

(a) Find bounds (lower, upper) on n.

(b) Construct the generating function.

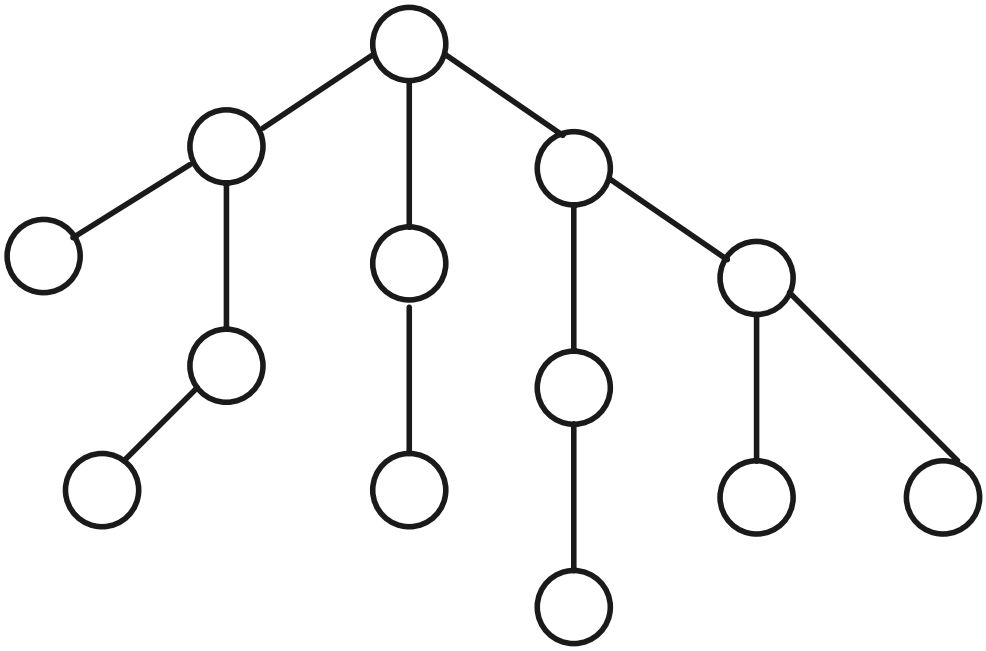
(c) Find the probabilities for n=5 and for n=11.

Q4. *Fibonacci numbers:*
Find an explicit formula for Fibonacci numbers:

$$F_0 = 0,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2},\ n = 2, 3, \cdots$$

Q5. *Tree traversal:*

(a) Find the bfs numbers and write in the nodes of the following tree:



(b) Find the dfs numbers and write in the nodes of the following tree: