Department of
Industrial Engineering

IE 454 Combinatorial Analysis

http://ie454.cankaya.edu.tr
Fall 2010 Tuesday 9:40-12:30 A201
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Solution to HOMEWORK 1

1. How many ways are there to arrange the letters in *ÇANKAYA*

   (a) with vowels in alphabetical order?

   $\binom{7}{3}$: spaces for vowels, 4!: consonants, 3!: Triple As $\quad \dfrac{\binom{7}{3}4!}{3!}$

   (b) with vowels together? 3!: within vowels, $\frac{5!}{3!}$: vowels and others $\quad \dfrac{3!\,5!}{3!} = 5!$

2. How many solutions are there to

   (a) $x_1 + x_2 + x_3 + x_4 = 11$, $x_i \geq 0$, integer. Multi-set problem, $n = 4$, $r = 11$: $\binom{11+4-1}{11} = \binom{14}{11}$

   (b) $x_1 + x_2 + \cdots + x_n = -r$, $x_i = 0$ or -1. Combination problem: $\binom{n}{r}$

3. What is the probability that

   (a) four rolls of a single die produces at least one six? $\frac{6^4-5^4}{6^4} = 0.5177$

   (b) twenty four rolls of a pair of distinct dice produces at least one *düşeş* (6 and 6)? $\frac{36^{24}-35^{24}}{36^{24}} = 0.4914$

   (c) a roll of three distinct dice produces a sum of eleven? *solved in class*

4. Intel produces 10 000 new chips. A sample of 85 is taken and 6 are found to be defective. What is the probability that this occurs if there are k defective chips?

$$\frac{\binom{k}{6}\binom{10000-k}{79}}{\binom{10000}{85}}$$

5. Prove the following from combinatorial viewpoint?

   (a) $\binom{n}{r} = \binom{n}{n-r}$.

   $\binom{n}{r}$ denotes the number of different ways of selecting r objects out of n objects in an urn. If we look at the same phenomenon from the viewpoint of the objects left in the urn, the number of different ways of selecting $n-r$ objects out of n is $\binom{n}{n-r}$. These two must be equal since we derive them from two viewpoints of the same phenomenon.

   (b) $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.

   $\binom{n}{r}$ denotes the number of different ways of selecting r balls out of n objects in an urn. Let us fix a ball, call it *super ball*. Two mutually exclusive alternatives exist; we either select the super ball or it stays in the urn. Given that the super ball is selected, the number of different ways of choosing $r-1$ balls out of $n-1$ is $\binom{n-1}{r-1}$. In the case that the super ball is not selected, $\binom{n-1}{r}$ denotes the number of ways of choosing r balls out of $n-1$. By the rule of sum, the right hand side is equal to the left hand side.

   (c) $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$.

   2^n is the number of subsets of a set of size n. $\binom{n}{0} = 1$ is for the empty set, $\binom{n}{n} = 1$ is for the set itself, and $\binom{n}{r}$, $r = 2, \ldots, n-1$ is the number of proper subsets of size r.

   (d) $\binom{n}{m}\binom{m}{r} = \binom{n}{r}\binom{n-r}{m-r}$.

   $\binom{n}{m}$ denotes the number of different ways of selecting m Industrial Engineering (IE) students out of n Çankaya students and $\binom{m}{r}$ denotes the number of different ways of selecting r IE students taking the Combinatorial Analysis (CA) course out of m IE students. On the other hand, $\binom{n}{r}$ denotes the number of ways of selecting r IE students taking CA from among n Çankaya students and $\binom{n-r}{m-r}$ denotes the number of different ways of selecting $(m-r)$ IE students not taking CA out of $(n-r)$ Çankaya students not taking CA. These two are equivalent.

   (e) $\binom{n}{0} + \binom{n+1}{1} + \cdots + \binom{n+r}{r} = \binom{n+r+1}{r}$.

   The right hand side, $\binom{n+r+1}{r}$, denotes the number of different ways of selecting r balls out of $m = n+2$ balls with repetition, known as the multi-set problem. Let us fix the super ball again. The left hand side is the list of the number of times that the super ball is selected in the above multi-set problem instance. That is, $\binom{n}{0}$ refers to the case in which the super ball is not selected, $\binom{n+1}{1}$ refers to the case in which the super ball is selected once, and $\binom{n+r}{r}$ refers to the case in which the super ball is always selected.