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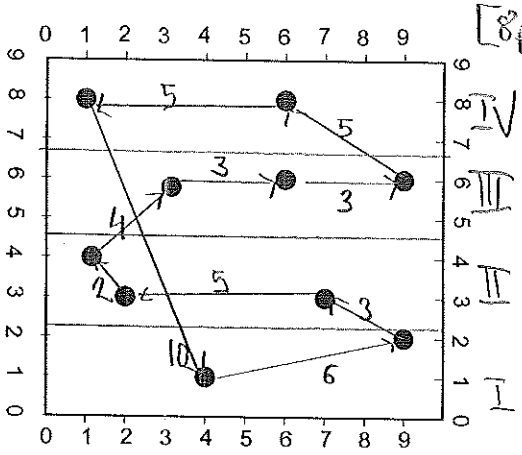
Tour: [80pt]

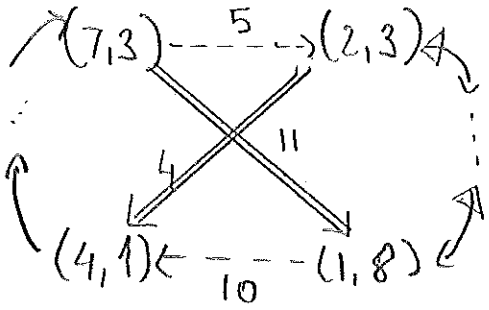
Q1. Apply the band heuristic with 4 horizontal strips for finding an initial TSP tour of the • points to be visited. Show the tour in the figure. Calculate the tour length in terms of Rectilinear Distance. Calculate the gain of deleting the edge between (7,3)–(2,3) and the edge between (1,8)–(4,1). What is your decision for this swap? Why? [80pt]



[8pt]

Length: 6+3+5+2+4+3+3+5+5+10 = 46 [2pt]

[20pt]



[10pt]

Gain: (10+5) − (11+4) = 0

[10pt] { Indifferent to make the move
still tour length is 46

- Wrong metric : -10pt
- Reverse direction : -2pt
- Not direct gain calculation : -5pt
- No final arc : -2pt

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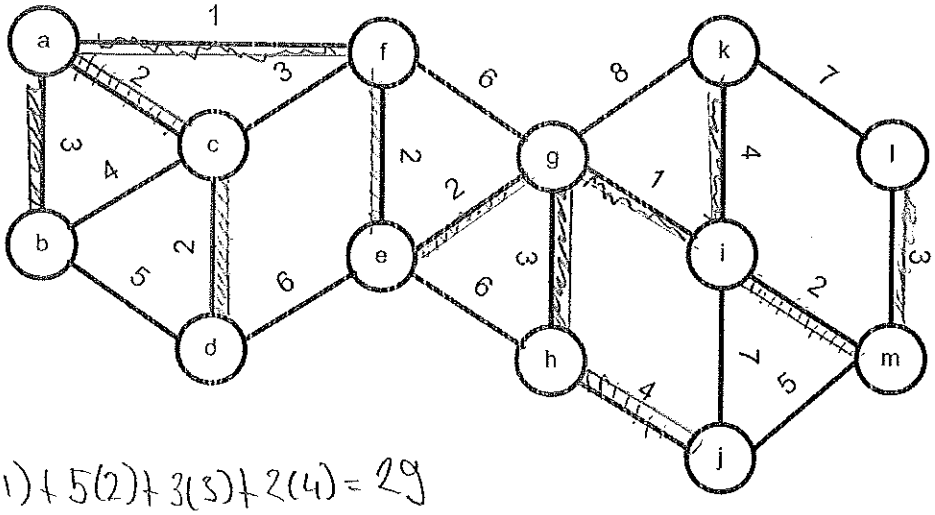
Q2.[40pt] Find the minimum spanning tree and the associated cost(length).



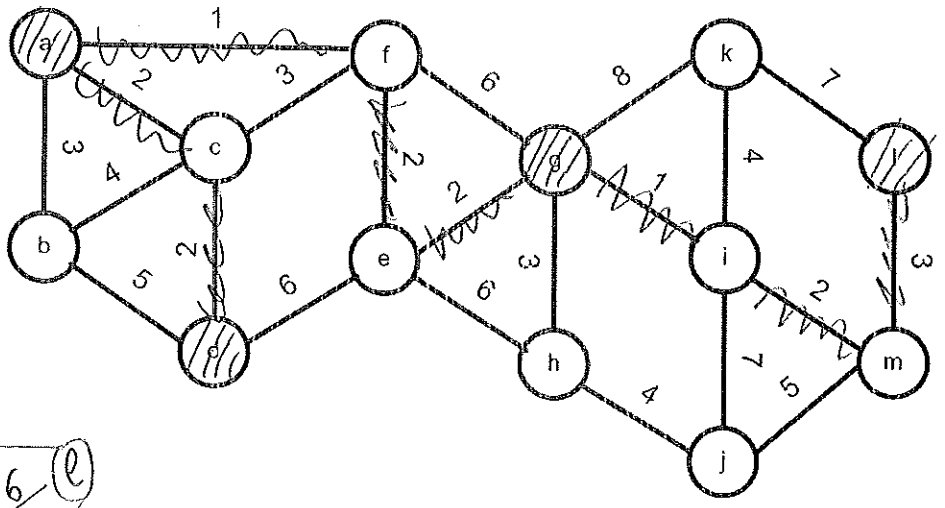
Tree Length: 2(1)+5(2)+3(3)+2(4) = 29

[4pt]

Each leg: 3pt

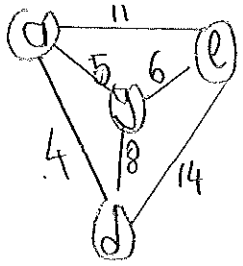
[36pt]

[40pt]Find a solution for the Steiner tree problem and the associated cost(length) where the selected set is $R = \{a, d, g, l\}$.



[18pt]

G1



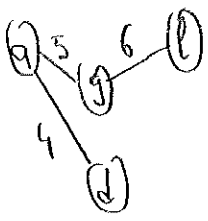
[3pt]

G3 = G4

S-Tree Length = 15

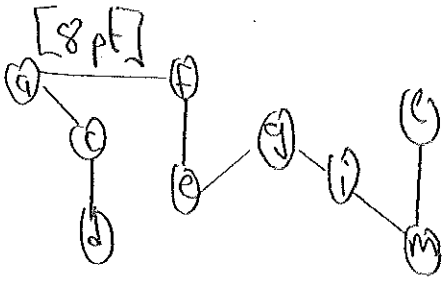
[2pt]

G2:



[9pt]

[8pt]



#arcs: -20pt.

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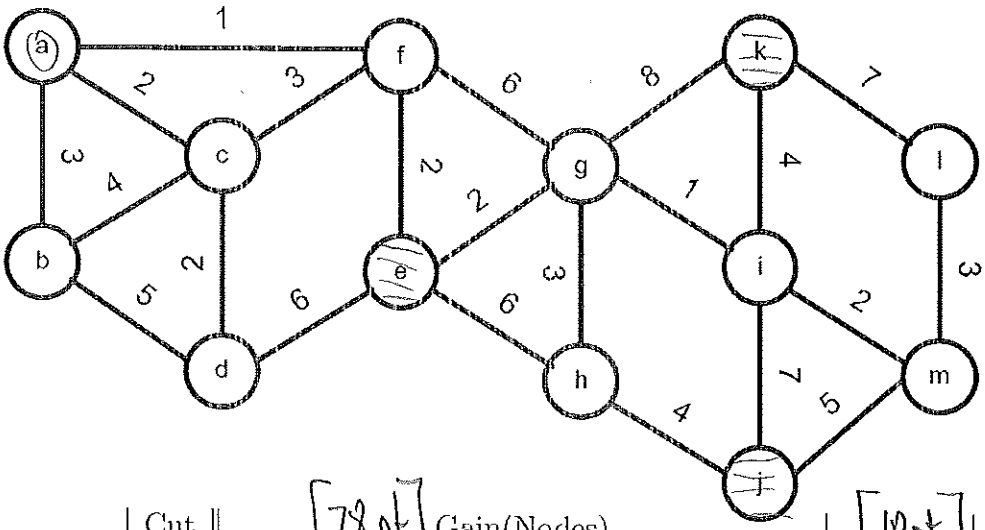
Q3. Apply the nongreedy method with threshold equal to 2 for finding the maximum cut (total weight of the edges in the cut defined by $S \subseteq \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$) problem for the following graph. Once you have moved a node, freeze it. Let the initial set be $S = \{a, e, j, k\}$. Break the ties by selecting the node with the minimum lexicographical order. Report the best greedy and nongreedy solutions. [100pt]



[78 pt]

Iter #	S =?	Cut Size	Gain(Nodes) a	b	c	d	e	f	g	h	i	j	k	l	m	[10pt] Decision	New cut size
1	{a e j k}	57	-6	6	7	1	-16	6	0	-7	-8	-16	-19	-4	0	c	64
2	{a c e j k}	64	-2	-2	[illegible]	-3	-16	0	0	-7	-8	-16	-19	-4	0	f	64
3	{a c e f j k}	64	0	-2	[illegible]	-3	-12	[illegible]	-12	-7	-8	-16	-19	-4	0	a	64
4	{c e f j k}	64	[illegible]	4	[illegible]	-3	-12	[illegible]	-12	-7	-8	-16	-13	-4	0	b	68
5	{b c e f j k}	68	[illegible]	[illegible]	[illegible]	-13	-12	[illegible]	-12	-7	-8	-16	-19	-4	0	m	68
6	{b c e f j k m}	68	[illegible]	[illegible]	[illegible]	-13	-12	[illegible]	-12	-7	-12	-6	-19	-10	[illegible]	STOP	
7	{ }																
8	{ }																
9	{ }																
10	{ }																
11	{ }																
12	{ }																
13	{ }																

Greedy stops → row 1

Nongreedy stops → row 6

[4pt] Greedy Solution: $S = \{a, c, e, j, k\}$

Max-cut Value: 64

[8pt] Nongreedy Solution:

$S_1 = \{b, c, e, f, j, k\}$

Max-cut Value: 68

$S_2 = \{b, c, e, f, j, k, m\}$ [2pt] if missing

Wrong decision -4pt

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Q4. Use dynamic programming to solve the following knapsack problem. Four categories of jobs are available, with quantities, times, and values shown in the table. We have 9 days available.

Job	Possible #	Time per job	Value per job
1	4	1	6
2	3	3	25
3	3	2	15
4	2	4	30

d_n: # jobs in category n selected,

x_n: # days remaining when we reach n.

Transformation functions:[5pt]

$x_3 = t_4(x_4, d_4) = ?\ x_4 - 4d_4$
$x_2 = t_3(x_3, d_3) = ?\ x_3 - 2d_3$
$x_1 = t_2(x_2, d_2) = ?\ x_2 - 3d_2$
$x_0 = t_1(x_1, d_1) = ?\ x_1 - d_1$

Return functions:[5pt]

$r_4(x_4, d_4) = ?\ 30d_4$
$r_3(x_3, d_3) = ?\ 15d_3$
$r_2(x_2, d_2) = ?\ 25d_2$
$r_1(x_1, d_1) = ?\ 6d_1$

Stage 1:[10pt]

x_1	d_1^*	$f_1(x_1)$
0	0	0
1	1	6
2	2	12
3	3	18
4	4	24
5	4	24
6	4	24
7	4	24
8	4	24
9	4	24

Stage 2:[15pt]

d_2:	$r_2(x_2, d_2) + f_1(x_1)$							
x_2	0	1	2	3		d_2^*	f_2^*	x_1
0	0+0					0	0	0
1	0+6					0	6	1
2	0+12					0	12	2
3	0+18	25+0				1	25	0
4	0+24	25+6				1	31	1
5	0+24	25+12				1	37	2
6	0+24	25+18	50+0			2	50	0
7	0+24	25+24	50+6			2	56	1
8	0+24	25+24	50+12			2	62	2
9	0+24	25+24	50+18	75+0		3	75	0

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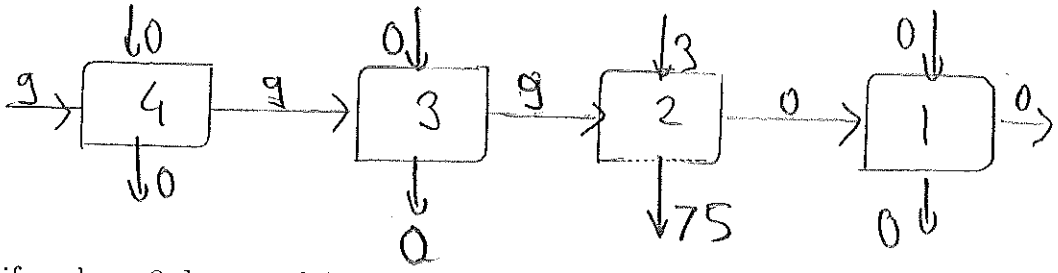
Stage 3: [15pt]

d_3:	$r_3(x_3, d_3) + f_2(x_2)$							
x_3	0	1	2	3	~~4~~	d_3^*	f_3^*	x_2
0	0+0					0	0	0
1	0+6					0	6	1
2	0+12	15+0				1	15	0
3	0+25	15+6				0	25	3
4	0+31	15+12	30+0			0	31	4
5	0+37	15+25	30+6			1	40	3
6	0+50	15+31	30+12	45+0		0	50	6
7	0+56	15+37	30+25	45+6		0	56	7
8	0+62	15+50	30+31	45+12		1	65	6
9	0+75	15+56	30+37	45+25		0	75	9

Stage 4:[10pt]

d_4:	$r_4(x_4, d_4) + f_3(x_3)$							
x_4	0	1	2	~~3~~	~~4~~	d_4^*	f_4^*	x_3
9	0+75	30+0	60+6	—	—	0	75	9

OPTIMUM SOLUTION (in box notation please): [10pt]



What if we have 8 days available? Please answer in box notation and indicate the total cost.

Stage 4: [10pt]

d_4:	$r_4(x_4, d_4) + f_3(x_3)$							
x_4	0	1	2	3	4	d_4^*	f_4^*	x_3
8	0+65	30+31	60+0	—	—	0	65	8

Box Notation: [10pt]