



Department of  
Industrial Engineering

## IE 454 Combinatorial Analysis

<http://ie454.cankaya.edu.tr>

Spring 2010 Tuesday 9:40-12:30 A201

*Levent Kandiller*

kandiller@cankaya.edu.tr

Voice: 189 Dean's office

### FINAL EXAM

1 page 2 sides self hand-written cheat sheet is allowed!

4.6.2010 - 17:00

*I hereby declare that I have neither given nor received any aid during the final exam.*

*I accept*

*I decline*

*Signature:*

Name: \_\_\_\_\_

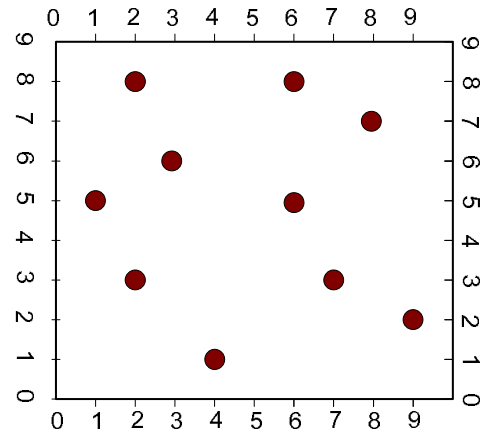
Surname: \_\_\_\_\_

Number: \_\_\_\_\_

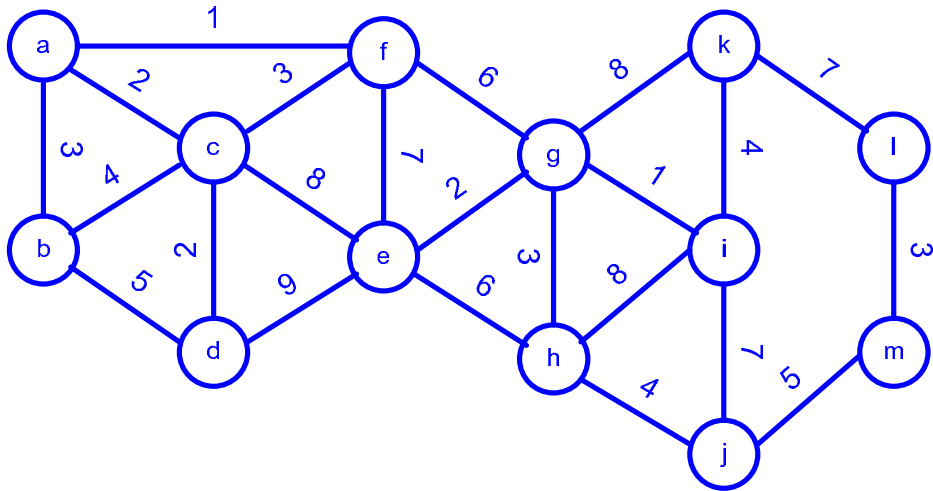
| Question | Time | Points | Obtained |
|----------|------|--------|----------|
| Q1       | 25   | 80     |          |
| Q2       | 15   | 80     |          |
| Q3       | 35   | 100    |          |
| Q4       | 35   | 90     |          |
| TOTAL    | 110  | 350    |          |

**GOOD LUCK!**

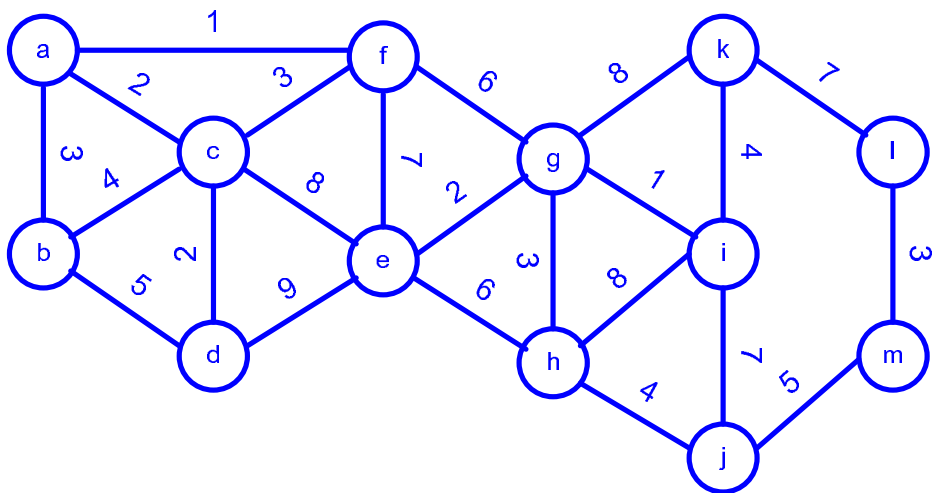
Apply the band heuristic with 4 vertical strips for finding an initial TSP tour of the • points to be visited. Show the tour in the figure. Calculate the tour length in terms of Rectilinear Distance. Calculate the gain of deleting the edge between (9,2)–(2,3) and the edge between (4,1)–(7,3). What is your decision for this swap? Why? [80pt]



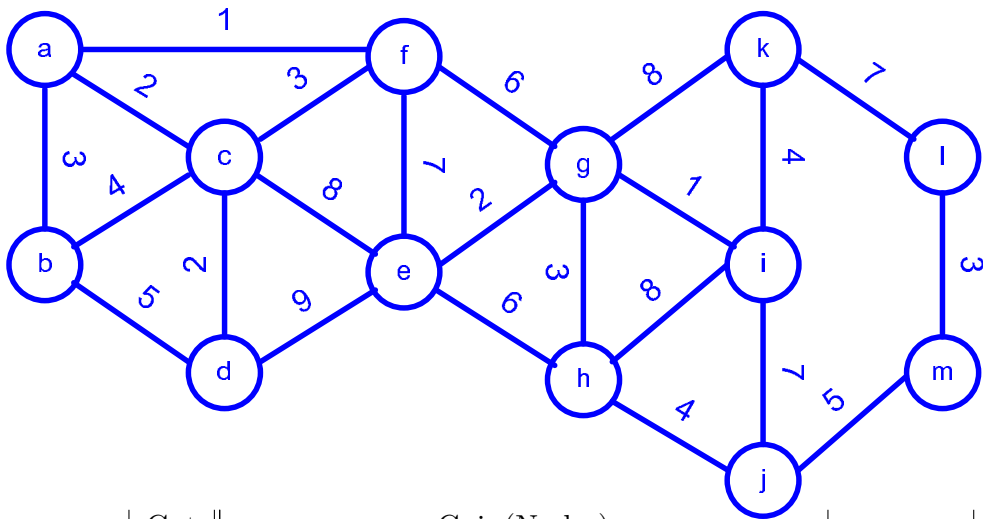
Q2.[40pt ] Find the minimum spanning tree and the associated cost(length).



[40pt ] Find a solution for the Steiner tree problem and the associated cost(length) where the selected set is  $R = \{a, d, g, m\}$ .



Q3. Apply the nongreedy method with threshold equal to 3 for finding the maximum cut (total weight of the edges in the cut defined by  $S \subseteq \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$ ) problem for the following graph. Once you have moved a node, freeze it. Let the initial set be  $S = \{c, h, k\}$ . Break the ties by selecting the node with the minimum lexicographical order. Report the best greedy and nongreedy solutions. [100pt]



| Iter # | S =? | Cut Size | Gain(Nodes) |   |   |   |   |   |   |   |   |   |   |   |   | Decision | New cut size |  |
|--------|------|----------|-------------|---|---|---|---|---|---|---|---|---|---|---|---|----------|--------------|--|
|        |      |          | a           | b | c | d | e | f | g | h | i | j | k | l | m |          |              |  |
| 1      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 2      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 3      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 4      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 5      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 6      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 7      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 8      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 9      | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 10     | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 11     | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 12     | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |
| 13     | { }  |          |             |   |   |   |   |   |   |   |   |   |   |   |   |          |              |  |

Greedy Solution:

Max-cut Value:

Nongreedy Solution:

Max-cut Value:

Q4. Use dynamic programming to solve the following knapsack problem. Four categories of jobs are available, with quantities, times, and values shown in the table. We have 9 days available.

| Job      | Possible # | Time per job | Value per job |
|----------|------------|--------------|---------------|
| <b>1</b> | 4          | 1            | 3             |
| <b>2</b> | 2          | 4            | 15            |
| <b>3</b> | 3          | 2            | 8             |
| <b>4</b> | 3          | 3            | 12            |

$d_n$ : # jobs in category  $n$  selected,

$x_n$ : # days remaining when we reach  $n$ .

Transformation

functions:[5pt]

$$x_3 = t_4(x_4, d_4) = ?$$

$$x_2 = t_3(x_3, d_3) = ?$$

$$x_1 = t_2(x_2, d_2) = ?$$

$$x_0 = t_1(x_1, d_1) = ?$$

Return functions:[5pt]

$$r_4(x_4, d_4) = ?$$

$$r_3(x_3, d_3) = ?$$

$$r_2(x_2, d_2) = ?$$

$$r_1(x_1, d_1) = ?$$

Stage 1:[10pt]

| $x_1$ | $d_1^*$ | $f_1(x_1)$ |
|-------|---------|------------|
| 0     |         |            |
| 1     |         |            |
| 2     |         |            |
| 3     |         |            |
| 4     |         |            |
| 5     |         |            |
| 6     |         |            |
| 7     |         |            |
| 8     |         |            |
| 9     |         |            |

Stage 2:[15pt]

| $d_2$ : | $r_2(x_2, d_2) + f_1(x_1)$ |   |   |   |   | $d_2^*$ | $f_2^*$ | $x_1$ |
|---------|----------------------------|---|---|---|---|---------|---------|-------|
| $x_2$   | 0                          | 1 | 2 | 3 | 4 |         |         |       |
| 0       |                            |   |   |   |   |         |         |       |
| 1       |                            |   |   |   |   |         |         |       |
| 2       |                            |   |   |   |   |         |         |       |
| 3       |                            |   |   |   |   |         |         |       |
| 4       |                            |   |   |   |   |         |         |       |
| 5       |                            |   |   |   |   |         |         |       |
| 6       |                            |   |   |   |   |         |         |       |
| 7       |                            |   |   |   |   |         |         |       |
| 8       |                            |   |   |   |   |         |         |       |
| 9       |                            |   |   |   |   |         |         |       |

Stage 3: [15pt]

| $d_3:$ | $r_3(x_3, d_3) + f_2(x_2)$ |   |   |   |   | $d_3^*$ | $f_3^*$ | $x_2$ |
|--------|----------------------------|---|---|---|---|---------|---------|-------|
| $x_3$  | 0                          | 1 | 2 | 3 | 4 |         |         |       |
| 0      |                            |   |   |   |   |         |         |       |
| 1      |                            |   |   |   |   |         |         |       |
| 2      |                            |   |   |   |   |         |         |       |
| 3      |                            |   |   |   |   |         |         |       |
| 4      |                            |   |   |   |   |         |         |       |
| 5      |                            |   |   |   |   |         |         |       |
| 6      |                            |   |   |   |   |         |         |       |
| 7      |                            |   |   |   |   |         |         |       |
| 8      |                            |   |   |   |   |         |         |       |
| 9      |                            |   |   |   |   |         |         |       |

Stage 4:[10pt]

| $d_4:$ | $r_4(x_4, d_4) + f_3(x_3)$ |   |   |   |   | $d_4^*$ | $f_4^*$ | $x_3$ |
|--------|----------------------------|---|---|---|---|---------|---------|-------|
| $x_4$  | 0                          | 1 | 2 | 3 | 4 |         |         |       |
| 9      |                            |   |   |   |   |         |         |       |

OPTIMUM SOLUTION (in box notation please): [10pt]

What if we have 8 days available? Please answer in box notation and indicate the total cost.

Stage 4: [10pt]

| $d_4:$ | $r_4(x_4, d_4) + f_3(x_3)$ |   |   |   |   | $d_4^*$ | $f_4^*$ | $x_3$ |
|--------|----------------------------|---|---|---|---|---------|---------|-------|
| $x_4$  | 0                          | 1 | 2 | 3 | 4 |         |         |       |
| 8      |                            |   |   |   |   |         |         |       |

Box Notation: [10pt]